Method of numerical solution of nonhydrostatic equations of compressible atmosphere in the weather forecast problem

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Our purpose:

- New nonhydrostatic semi–lagrangian semi–implicit finite–difference dynamical core for the SLAV model.

SLAV:

» is the global operational model in Russia (Tolstykh, 2010).

» Based on the equations of hydrothermodynamics for the incompressible atmosphere in hydrostatic equilibrium.

» Horizontal resolution for the Russian territory is assumed to grow by 2012 to 20km, whereas the number of vertical layers will be increased from 28 to 60.

Atmospherical equations

$$\frac{\mathrm{dU}}{\mathrm{d}t} + RT\nabla_{xy}\ln(p) + fk \times \mathrm{U} = 0$$

$$\frac{\mathrm{dw}}{\mathrm{d}t} + g + RT\nabla_{z}\ln(p) = 0$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} - T\frac{R}{c_p}\frac{\ln(p)}{\mathrm{d}t} = 0$$

$$\left(1 - \frac{R}{c_p}\right)\frac{\ln(p)}{\mathrm{d}t} + D_3 = 0$$

$$p\alpha = RT$$

Hydrostatic approach:

$$\frac{\mathrm{dw}}{\mathrm{d}t} \ll g$$

ref. hydrostatic state: $\nabla_z \ln p_* = -\frac{g}{RT_*(z)}$

$$(\ln p)' = \ln \frac{p}{p_*} \qquad P = RT_*(\ln p)'$$

$$T' = T - T_* \qquad B = g\frac{T}{T_*} - \frac{g}{c_p}T_*P$$

(Thomas S. et al, 1998)

$$(\ln p)' = \ln \frac{p}{p_*} \qquad P = RT_*(\ln p)'$$

$$T' = T - T_* \qquad B = g\frac{T}{T_*} - \frac{g}{c_p}T_*P$$

$$\begin{aligned}
&\frac{\mathrm{dU}}{\mathrm{d}t} + RT\nabla_{xy}\ln(p) + fk \times \mathrm{U} = 0 & & \frac{\mathrm{dU}}{\mathrm{d}t} + \nabla_{xy}P = R_{\mathrm{U}} \\
&\frac{\mathrm{dw}}{\mathrm{d}t} + g + RT\nabla_z \ln(p) = 0 & \longrightarrow \quad & \frac{\mathrm{dw}}{\mathrm{d}t} + \left(\nabla_z - \frac{N_*^2}{g}\right)P - B = R_{\mathrm{w}} \\
&\frac{\mathrm{d}T}{\mathrm{d}t} - T\frac{R}{c_p}\frac{\ln p}{\mathrm{d}t} = 0 & & \frac{\mathrm{d}B}{\mathrm{d}t} + \mathrm{w}N_*^2 = R_B \\
&\left(1 - \frac{R}{c_p}\right)\frac{\ln p}{\mathrm{d}t} + D_3 = 0 & & \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{c_*^2}\right) + D_3 - \frac{g}{c_*^2}\mathrm{w} = 0
\end{aligned}$$

♣ Left side contains material derivatives plus linear forcing terms associated with acoustic and gravity oscillations.

2-dim case (in vertical plane).
Terrain–following Z coordinate: $\nabla \rightarrow \delta$

$$
\begin{aligned}
&\frac{\mathrm{d}u}{\mathrm{d}t} + \nabla_x P = R_u && \frac{\mathrm{d}u}{\mathrm{d}t} + \delta_x P = R_u \\
&\frac{\mathrm{dw}}{\mathrm{d}t} + \left(\nabla_z - \frac{N_*^2}{g}\right) P - B = R_{\mathrm{w}} \longrightarrow && \frac{\mathrm{dw}}{\mathrm{d}t} + \left(\delta_z - \frac{N_*^2}{g}\right) P - B = R_{\mathrm{w}} \\
&\frac{\mathrm{d}B}{\mathrm{d}t} + \mathrm{w}N_*^2 = R_B && \frac{\mathrm{d}B}{\mathrm{d}t} + \mathrm{w}N_*^2 = R_B \\
&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{c_*^2}\right) + D_3 - \frac{g}{c_*^2}\mathrm{w} = 0 && \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{c_*^2}\right) + (\delta_x u + \delta_z \mathrm{w}) - \frac{g}{c_*^2}\mathrm{w} = 0
\end{aligned}
$$

Here are:

$$
\delta_x = \frac{\partial}{\partial x_Z} - \frac{G_1}{G_0}\frac{\partial}{\partial Z} \qquad G_1 = \frac{\partial z}{\partial x_Z}
$$

$$
\delta_z = \frac{1}{G_0}\frac{\partial}{\partial Z} \qquad G_0 = \frac{\partial z}{\partial Z}
$$

Semi–implicit scheme:

$$
\begin{aligned}
u^{n+1} &= -\Delta t_+ \delta_x P^{n+1} + \hat{u}^n \\
w^{n+1} &= -\Delta t_+ \left(\delta_z P^{n+1} - \frac{N_*^2}{g} P^{n+1} - B^{n+1} \right) + \hat{w}^n \\
B^{n+1} &= -\Delta t_+ N_*^2 w^{n+1} + \hat{B}^n \\
\frac{P^{n+1}}{c_*^2} &= -\Delta t_+ \left(\delta_x u^{n+1} + \delta_z w^{n+1} - \frac{g}{c_*^2} w^{n+1} \right) + \hat{P}^n
\end{aligned}
$$

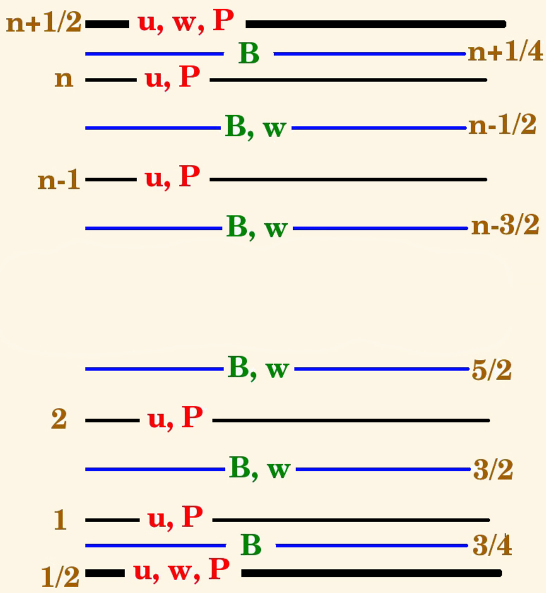
where $\hat{\cdot}$ values are known.
Main equation:

$$
\Delta t_+^2 \left(\delta_x^2 + e_0 \delta_z^2 \right) P - \left(\frac{N_*^2}{g} + \frac{g}{c_*^2} \right) \delta_z P + \left(\frac{N_*^2}{c_*^2} - \frac{1}{c_*^2} \right) P
$$

$$
= e_2 \left(\delta_z \hat{B} + \frac{1}{\Delta t_+} \delta_z \hat{w} - \frac{g}{c_*^2} \hat{B} - \frac{g}{c_*^2 \Delta t_+} \hat{w} \right) + \Delta t_+ \delta_x \hat{u} - \hat{P}
$$

$$
\begin{aligned}
e_2 &= \Delta t_\pm e_1 = \frac{(\Delta t_\pm)^2}{1 + \Delta t_+^2 N_*^2} \\
\Delta t_\pm &= \Delta t \frac{1 \pm \epsilon}{2}
\end{aligned}
$$

Vertical structure of the model.



Boundary condition:

- $u_{1/2} = u_1$
- $w_{n+1/2} = 0$

$$\delta_x^2 = \delta_{2x}\delta_x$$

where

$$\delta_x = \tilde{\delta}_x - G^{13}\mu_x\tilde{\delta}_Z$$

$$\delta_{x2} = \tilde{\delta}_x + \frac{\overline{g}^{13}}{H}\mu_x - \frac{1}{G^0}\tilde{\delta}_Z G^1\mu_x$$

so:

$$\delta_x^2 = \delta_{x2}\delta_x =$$

$$\tilde{\delta}_x^2 - \tilde{\delta}_x G^{13}\mu_x\tilde{\delta}_Z + \frac{\overline{g}^{13}}{H}\mu_x\tilde{\delta}_x - \frac{\overline{g}^{13}}{H}\mu_x G^{13}\mu_x\tilde{\delta}_Z$$

$$-\frac{1}{G^0}\tilde{\delta}_Z G^1\mu_x\tilde{\delta}_x + \frac{1}{G^0}\tilde{\delta}_Z G^1\mu_x G^{13}\mu_x\tilde{\delta}_Z$$

Equation on the surface.

Vertical velocity:

in **Z**: $W_{i,1/2} = 0$
in z: $w_{i,1/2} = G_{1\ i,1/2} u_{i,1/2} = G_{1\ i,1/2} u_{i,1}$

On the intermediate $k = 3/4$ level:

$$w_{i,3/4} = -e1(\delta_z P - N_*^2/gP)_{i,3/4} + e_1 \hat{B}_{i,3/4} + e_0 \hat{w}_{i,3/4}$$

at the same time

$$w_{i,3/2} = -e1(\delta_z P - N_*^2/gP)_{i,3/2} + e_1 \hat{B}_{i,3/2} + e_0 \hat{w}_{i,3/2}$$
$$w_{i,1/2} = G_{1\ i,1/2} u_{i,1} = G_{1\ i,1/2}(-\Delta t_+ (\delta_x P)_{i,1} + \hat{u}_{i,1})$$

linear interpolation scheme:

$$w_{i,3/4} = a w_{i,3/2} + b w_{i,1/2}$$

$\Rightarrow$ independent equation for $P_{i,1/2}$ based on the nonhydrostatic equation and surface boundary condition.

Equation near the top.

Top boundary condition:

$$w_{i,n+1/2} = W_{i,n+1/2} = 0$$

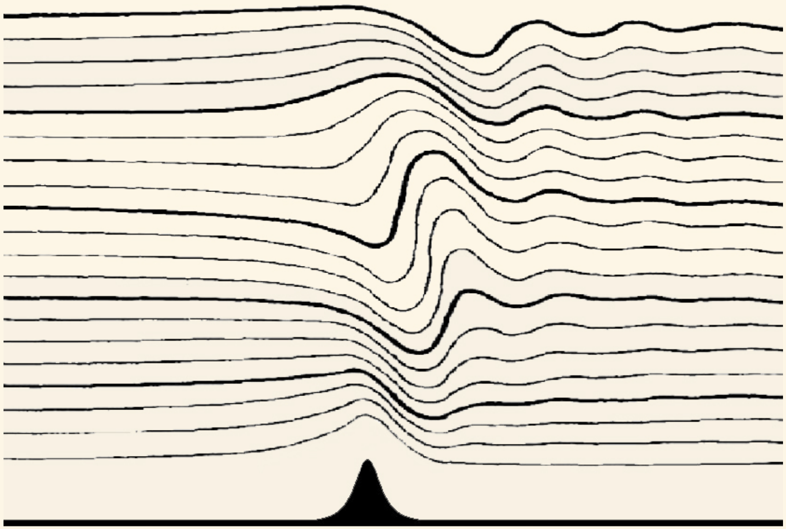
Hydrostatic approach on the $n + 1/4$ level:

$$(\delta_z \ln p)_{i,n+1/4} = -\frac{g}{RT_{i,n+1/4}}$$

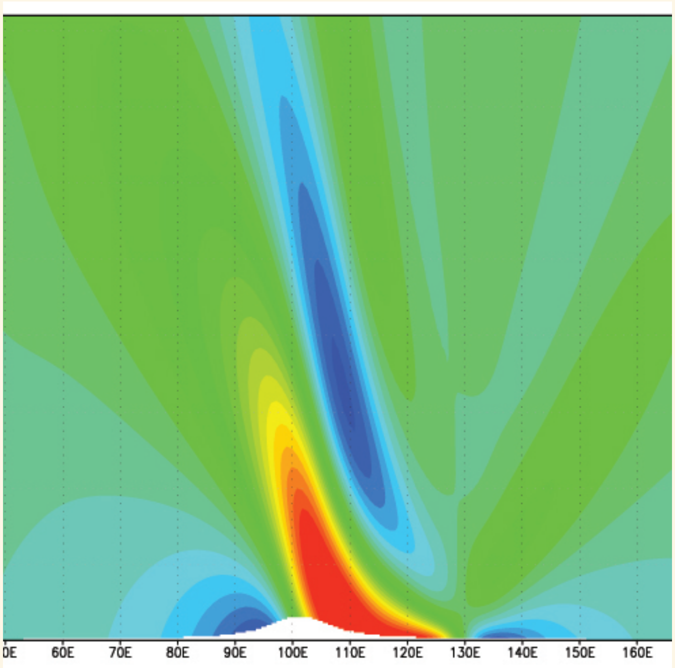
Equation for the $P_{i,n+1/4}$:

$$(\delta_z P)_{i,n+1/4} = g\frac{T_{i,n+1/4} - T_*}{T_{i,n+1/4}} = \frac{b_{i,n+1/4}}{1 + b_{i,n+1/4}/g}$$

Experiment: orographic waves.



velocity deviation (u).



- dx=550m, dz=150m
- u=10 m/s
- h=300m

Conclusion:

- We developed the numerical method of the solution of nonhydrostatic equations.
- Further we plan to construct 3-dim nonhydrostatic dynamical core for the SLAV model.

Thank you for attention!