Modification and implementation of the CABARET scheme in the Coupled Climate Model INM RAS

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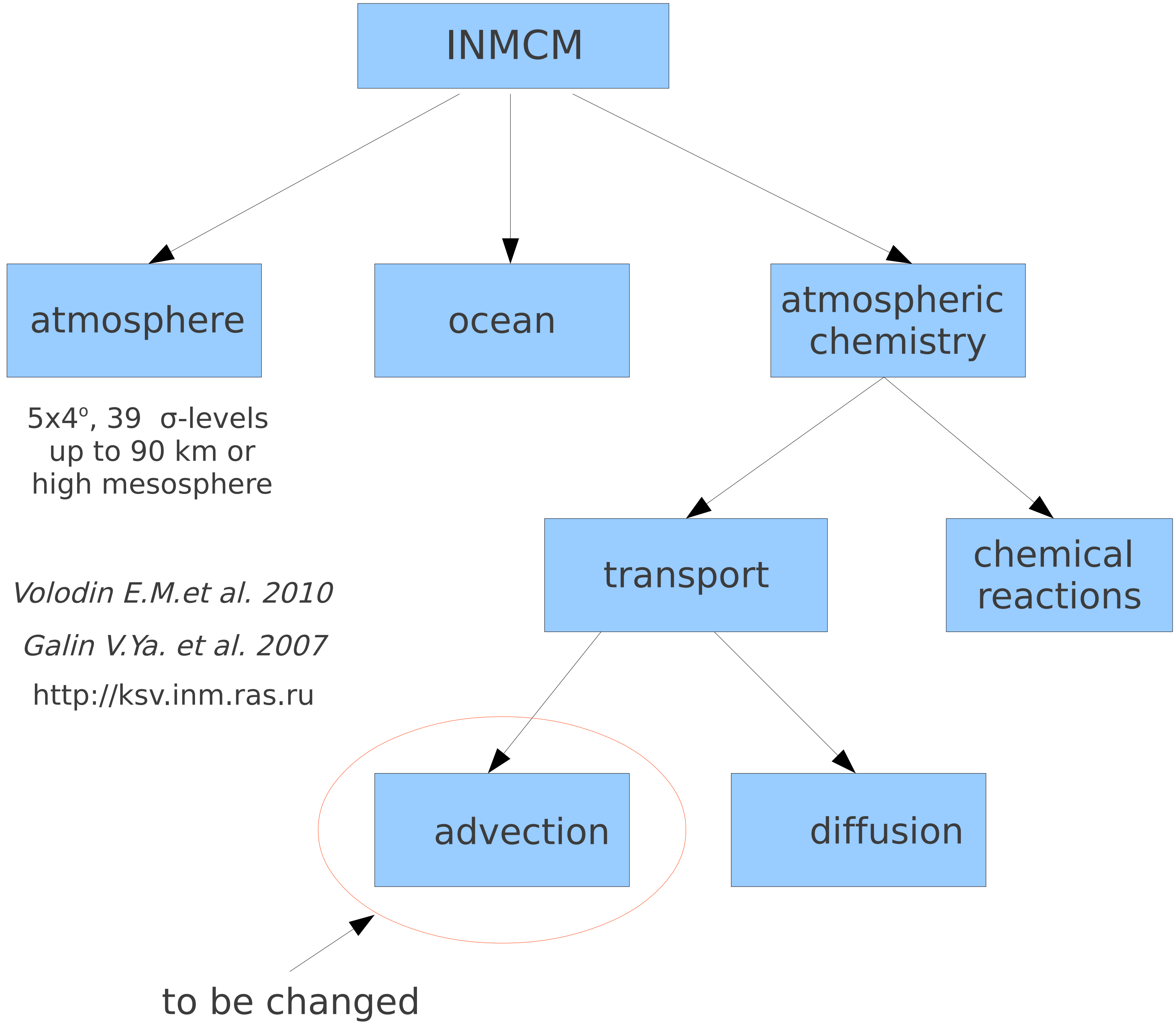
Coupled Climate Model of INM RAS

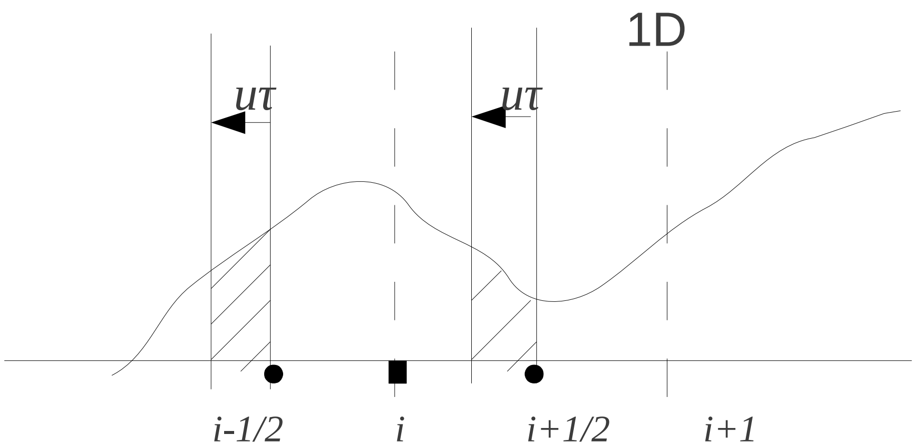


CABARET advection scheme in 1D case

CABARET – Compact Accurately Boundary-Adjusting high-REsolution Technique

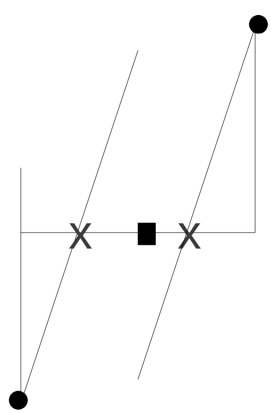
(upwind leapfrog)

Goloviznin et al., 2003



Flux form equation for conservative variables

$$\frac{\rho_i^{n+1}-\rho_i^n}{\tau}+\frac{(u\rho)_{i+\frac{1}{2}}^{n+\frac{1}{2}}-(u\rho)_{i-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x_i}=0$$

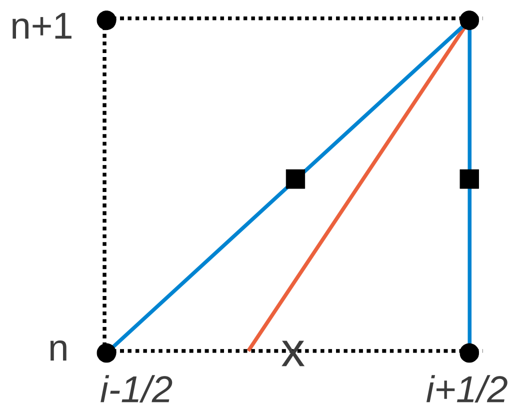


Interpolation of flux variables

$$\rho_{i+\frac{1}{2}}^{n+\frac{1}{2}}+\rho_{i-\frac{1}{2}}^{n-\frac{1}{2}}=2\rho_i^n$$

CABARET monotonization procedure

Goloviznin et al., 2003,,



$$\frac{d\rho}{dt} = f(x,t)$$

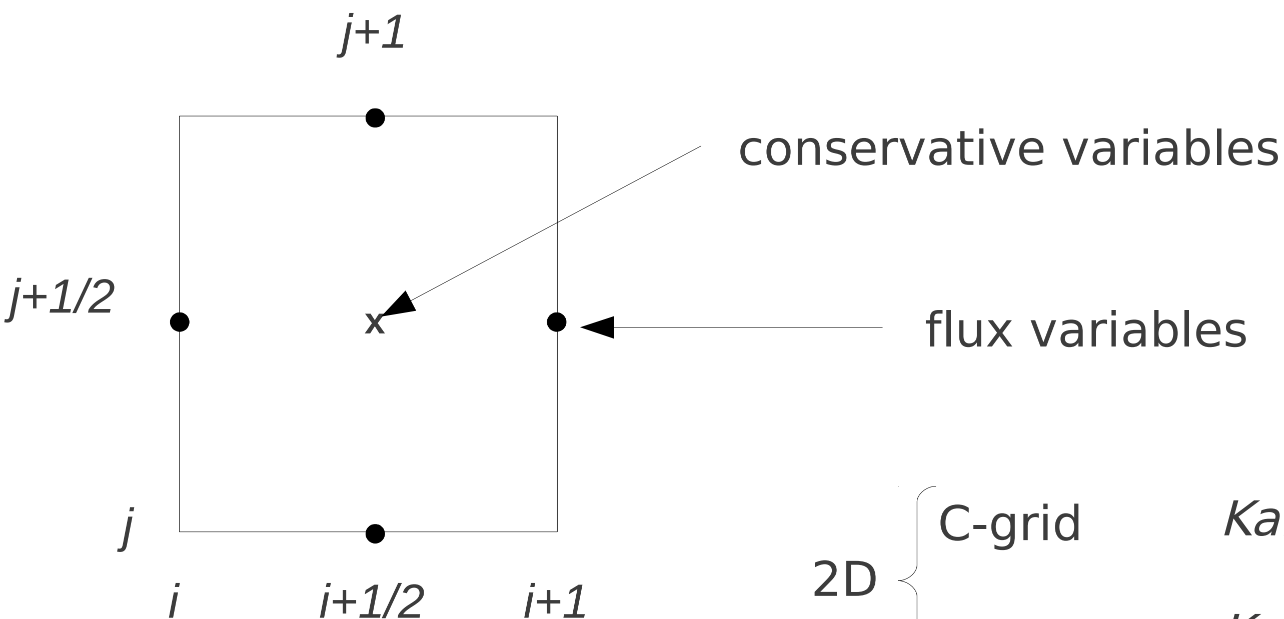
$$\rho_{i+1/2}^{n+1} = \rho_o^n + \int_{t^n}^{t^{n+1}} f(x(t),t)\,dt \approx \rho_o^n + f(x(t^{n+1/2}), t^{n+1/2})\tau$$

$$m_i^n = \rho_{i-1/2}^n + f(x_i, t^{n+1/2})\tau, M_i^n = \rho_{i+1/2}^n + f(x_{i+1/2}, t^{n+1/2})\tau$$

$$min(m_i, M_i) \le \rho_{i+1/2}^{n+1} \le max(m_i, M_i)$$

CABARET advection scheme in 2D case

$$\frac{\partial \rho}{\partial t}+\frac{\partial u \rho}{\partial x}+\frac{\partial v \rho}{\partial y}=0$$



2D
- C-grid — *Karabasov, Goloviznin 2007*
- B-grid — *Kostrykin 2010*

CABARET advection scheme in 2D case

I. Update conservative variables

$$\rho^{n+1/2}_{i+1/2,\,j+1/2} = \rho^{n-1/2}_{i+1/2,\,j+1/2} - \tilde{u}\left(\rho^{n}_{i+1,\,j+1/2} - \rho^{n}_{i,\,j+1/2}\right) - \tilde{v}\left(\rho^{n}_{i+1/2,\,j+1} - \rho^{n}_{i+1/2,\,j}\right)$$

II. Update flux variables

$$\hat{\rho}^{n+1}_{i+1,\,j+1/2} = 2\rho^{n+1/2}_{i+1/2,\,j+1/2} - \rho^{n}_{i,\,j+1/2},\ \hat{\rho}^{n+1}_{i+1/2,\,j+1} = 2\rho^{n+1/2}_{i+1/2,\,j+1/2} - \rho^{n}_{i+1/2,\,j}$$

III. Flux variables correction

$$m_{i+1,\,j+1/2} = min\left(\rho^{n}_{i+1,\,j+1/2}, \rho^{n}_{i,\,j+1/2}\right), M_{i+1,\,j+1/2} = max\left(\rho^{n}_{i+1,\,j+1/2}, \rho^{n}_{i,\,j+1/2}\right),$$

$$m_{i+1/2,\,j+1} = min\left(\rho^{n}_{i+1/2,\,j+1}, \rho^{n}_{i+1/2,\,j}\right), M_{i+1/2,\,j+1} = max\left(\rho^{n}_{i+1/2,\,j+1}, \rho^{n}_{i+1/2,\,j}\right),$$

$$\rho^{n+1}_{i+1,\,j+1/2} = max\left(min\left(\hat{\rho}^{n+1}_{i+1,\,j+1/2}, M_{i+1,\,j+1/2}\right), m_{i+1,\,j+1/2}\right),$$

$$\rho^{n+1}_{i+1/2,\,j+1} = max\left(min\left(\hat{\rho}^{n+1}_{i+1/2,\,j+1}, M_{i+1/2,\,j+1}\right), m_{i+1/2,\,j+1}\right)$$

Monotonicity of the CABARET scheme

1D *Ostapenko, 2009*

2D,3D **?**

IIIa

$$m_{i+1,j+1/2}=min(\rho^n_{i+1,j+1/2},\rho^n_{i,j+1/2}),\ M_{i+1,j+1/2}=max(\rho^n_{i+1,j+1/2},\rho^n_{i,j+1/2}),$$
$$m_{i+1/2,j+1}=min(\rho^n_{i+1/2,j+1},\rho^n_{i+1/2,j}),\ M_{i+1/2,j+1}=max(\rho^n_{i+1/2,j+1},\rho^n_{i+1/2,j}),$$
$$\rho^{n+1}_{i+1,j+1/2}=max(0.,max(min(\hat{\rho}^{n+1}_{i+1,j+1/2},M_{i+1,j+1/2}),m_{i+1,j+1/2})),$$
$$\rho^{n+1}_{i+1/2,j+1}=max(0.,max(min(\hat{\rho}^{n+1}_{i+1/2,j+1},M_{i+1/2,j+1}),m_{i+1/2,j+1}))$$

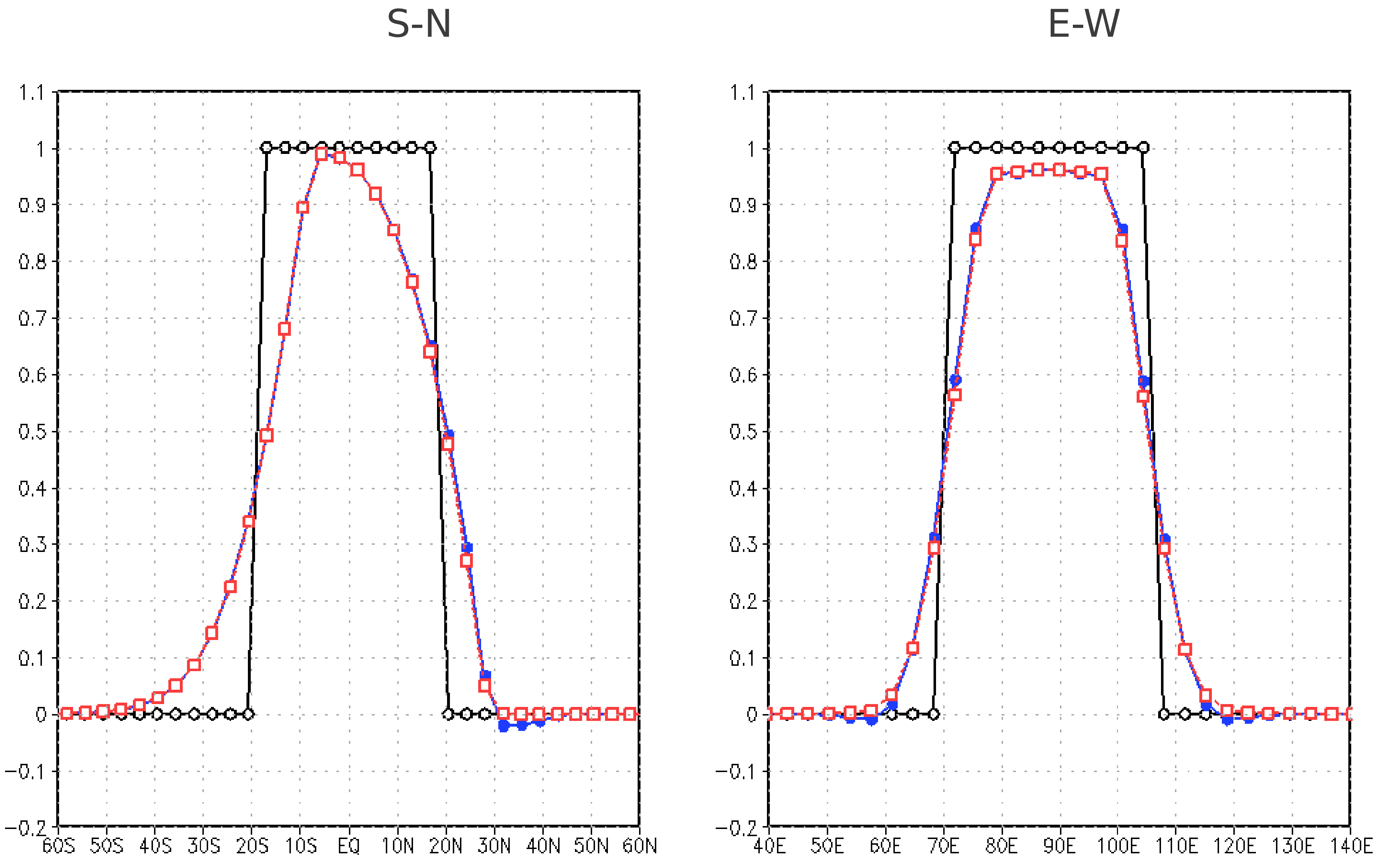
Statement. Under condition $\tilde{u}\leq\frac{1}{4},\tilde{v}\leq\frac{1}{4}$ scheme I, II, IIIa is positive.

Proof

$$\rho^{n+1/2}_{i+1/2,j+1/2}\geq\rho^{n-1/2}_{i+1/2,j+1/2}-u\rho^n_{i+1,j+1/2}-v\rho^n_{i+1/2,j+1}$$
$$\rho^{n-1/2}_{i+1/2,j+1/2}=\frac{1}{2}(\rho^{n-1}_{i,j+1/2}+\rho^n_{i+1,j+1/2}),$$
$$\rho^{n-1/2}_{i+1/2,j+1/2}=\frac{1}{2}(\rho^{n-1}_{i+1/2,j}+\rho^n_{i+1/2,j+1}),$$
$$\rho^{n+1/2}_{i+1/2,j+1/2}\geq(\frac{1}{4}-\tilde{u})\rho^n_{i+1,j+1/2}+\tilde{u}\rho^{n-1}_{i,j+1/2}+(\frac{1}{4}-\tilde{v})\rho^n_{i+1/2,j+1}+\tilde{v}\rho^{n-1}_{i+1/2,j}.$$

Results of the solid-body rotation test on the sphere

$$\alpha = \frac{\pi}{2}$$



Problem of pressure-tracer inconsistency

$$\frac{\partial \pi q}{\partial t} + div(\pi q \boldsymbol{u}) = 0, (1)$$

$$\frac{\partial \pi}{\partial t} + div(\pi \boldsymbol{u}) = 0, (2)$$

$$\frac{\partial \pi}{\partial t} + \int_0^1 div_2(\pi \boldsymbol{u})\, d\sigma = 0 (3)$$

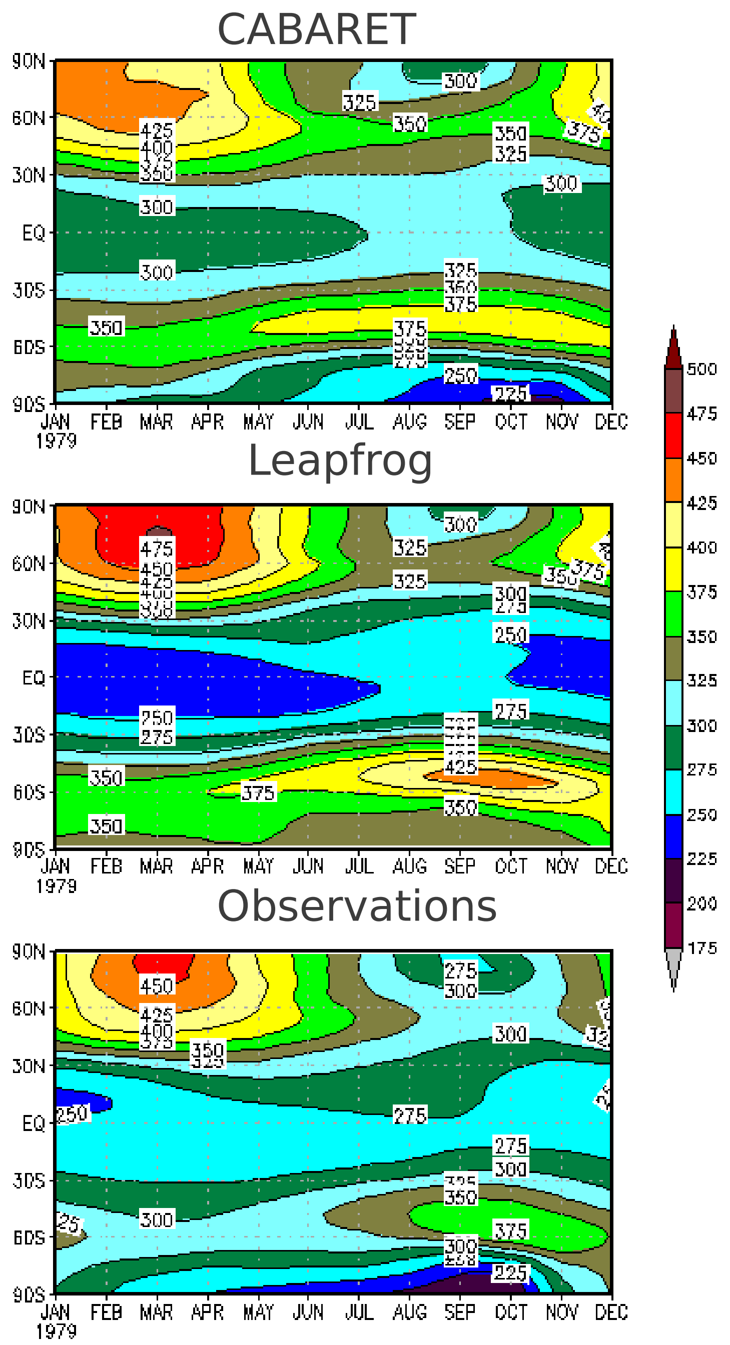
$$\frac{(\pi q)^{n+1} - (\pi q)^n}{\Delta t} + F^{n+1/2}(\pi q \boldsymbol{u}) = 0, (4)$$

$$\frac{\pi^{n+1} - \pi^n}{\Delta t} + F_2^{n+1/2}(\pi \boldsymbol{u}) + (\pi \dot{\sigma})_\sigma^{n+1/2} = 0, (5)$$

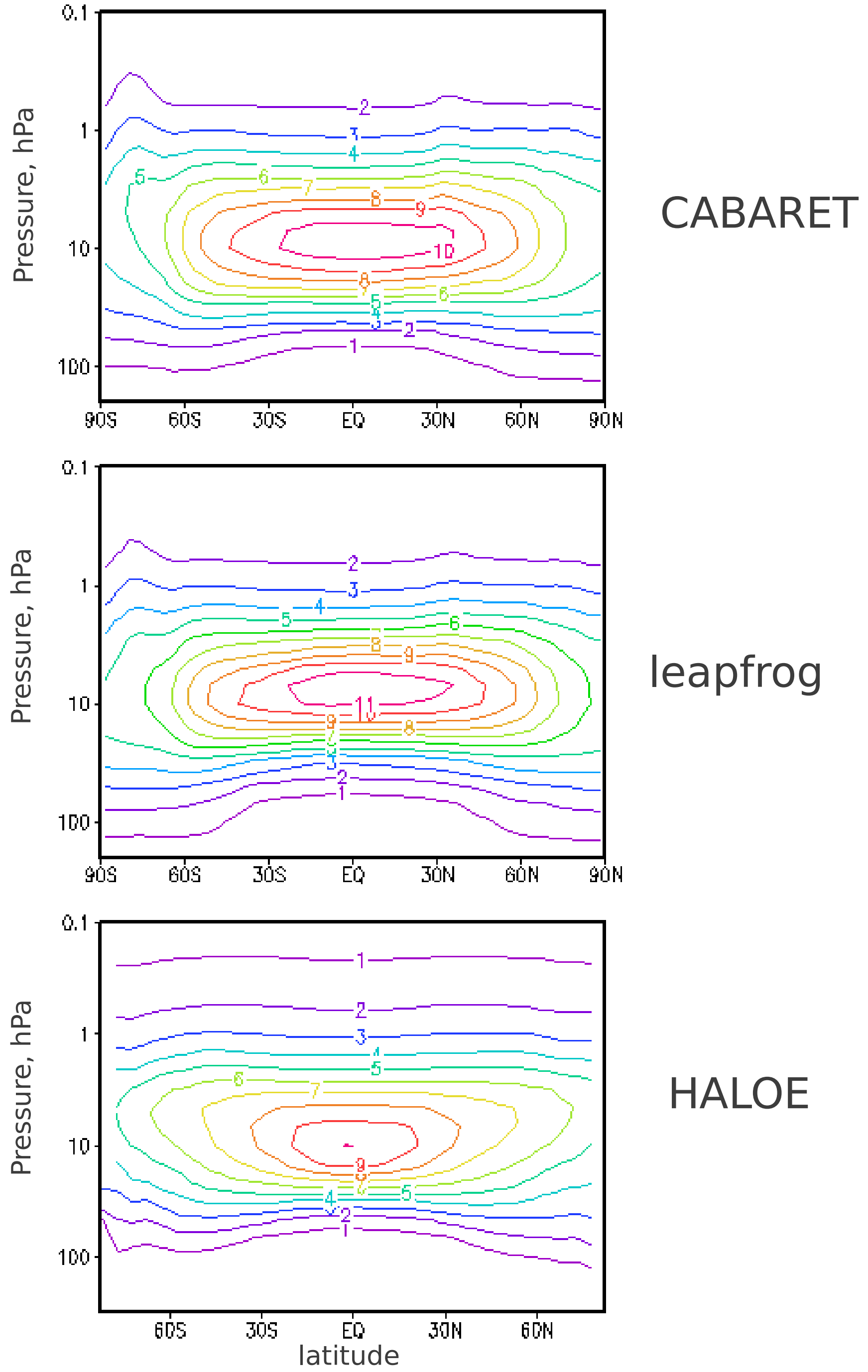
$$\frac{\pi^{n+1} - \pi^n}{\Delta t} + \sum_{i=1}^{N} F_2^{n+1/2}(\pi \boldsymbol{u}_i) = 0. (6)$$

1) For global conservation one should know the pressure field on the next time step before advection
2) Inconsistency in calculation of advection of tracer and pressure could lead to the non-monotonicity of the tracer distribution

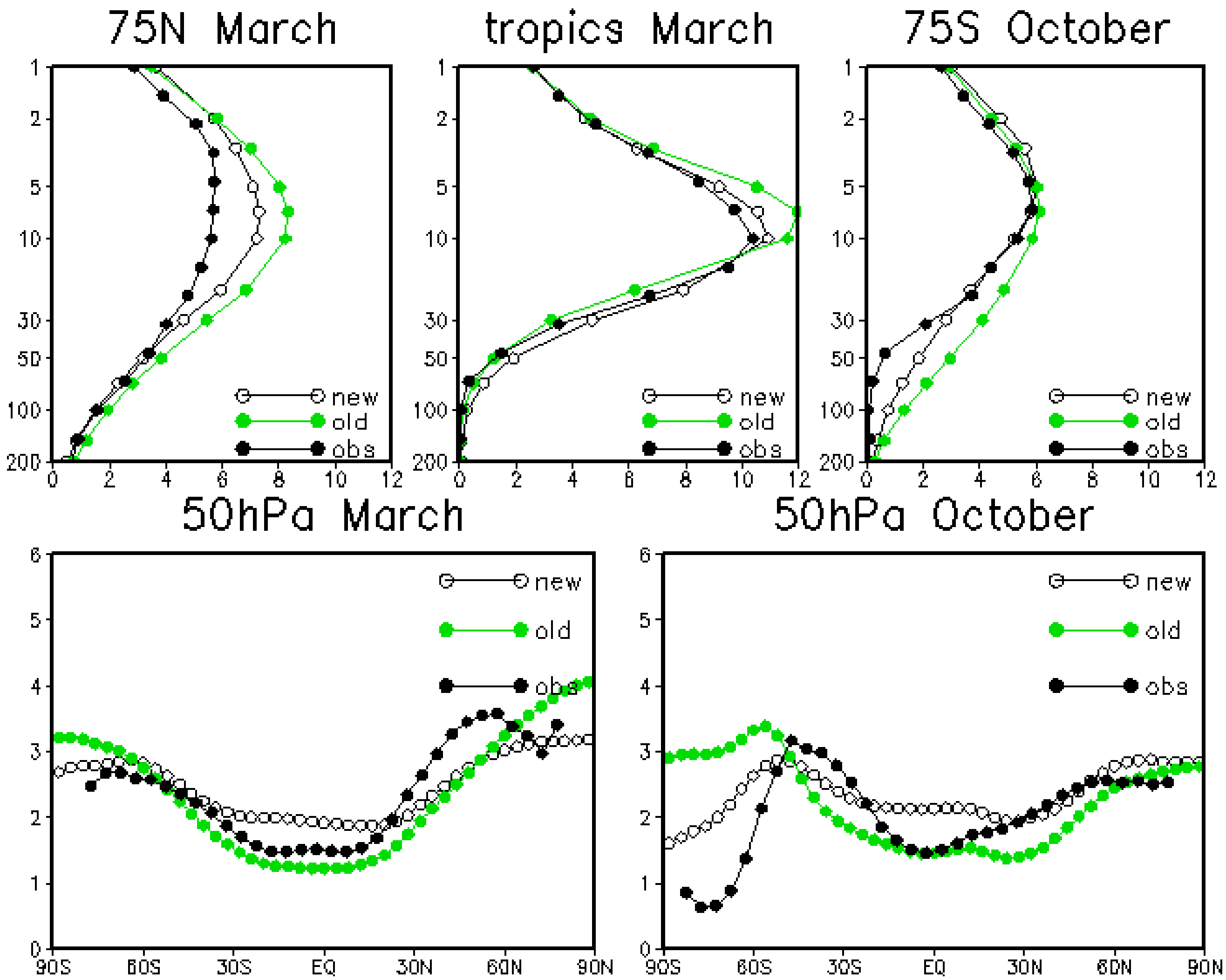
Seasonal cycle of total ozone column



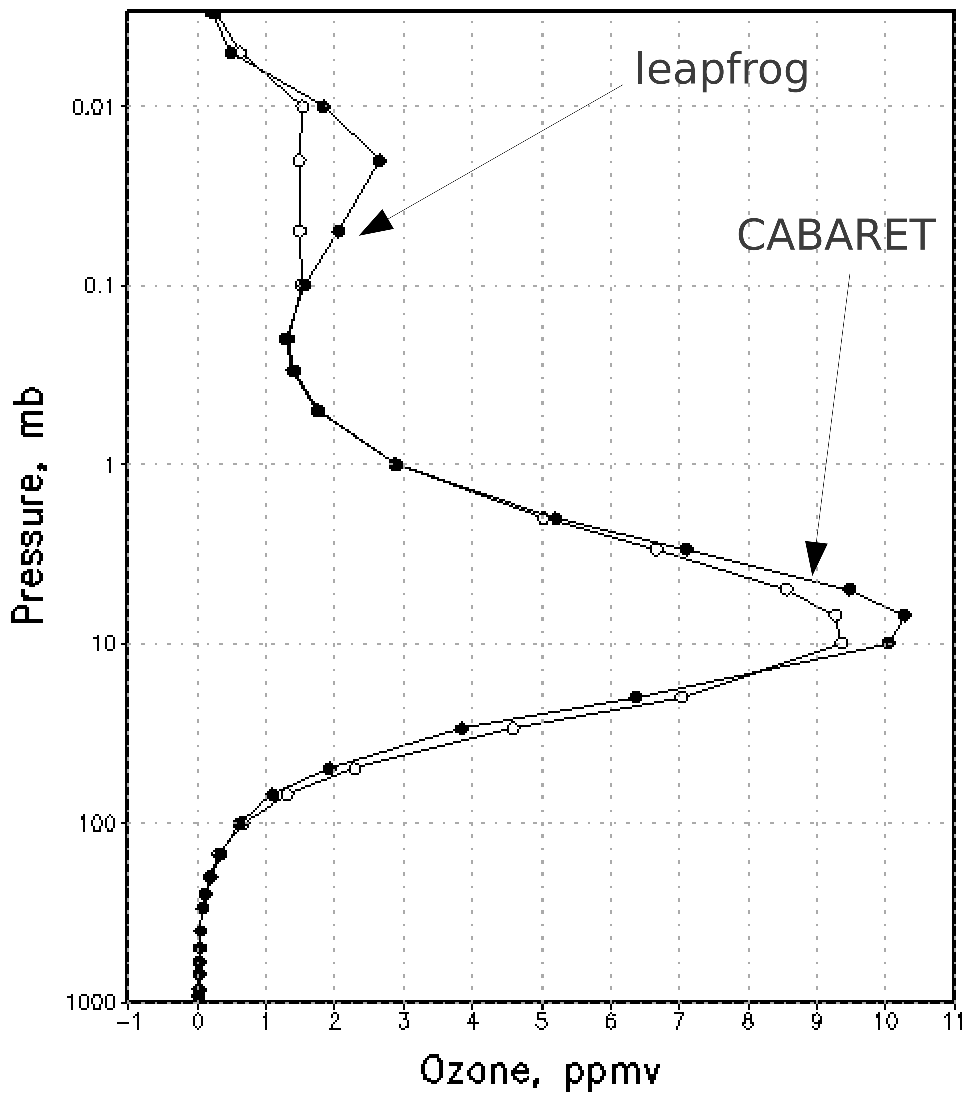
Annual distribution of Ozone (ppmv)



Vertical profiles ozone



Zonally mean annual ozone profile



Conclusions:

1) A positive multidimensional version of CABARET scheme is proposed
2) The implementation of the new advection schemes into chemical transport model leads to:
• improvement of ozone climatology near stratopause (disappear artificial maximum) and near stratospheric ozone maxima (decreasing)
• improvement in the description of total ozone column at high and middle latitudes SH (deeper ozone holes, weaker midlatitude maxima)
• at tropics a total ozone column becomes larger than in observations
3) Elapsed time for CTM increases on 30%. Memory storage increases in 4 times for each tracer.

Thanks for your attention!

Annual distribution of HCl (ppbv)