MODELS OF VOLCANIC ASH FIELDS RECONSTRUCTION

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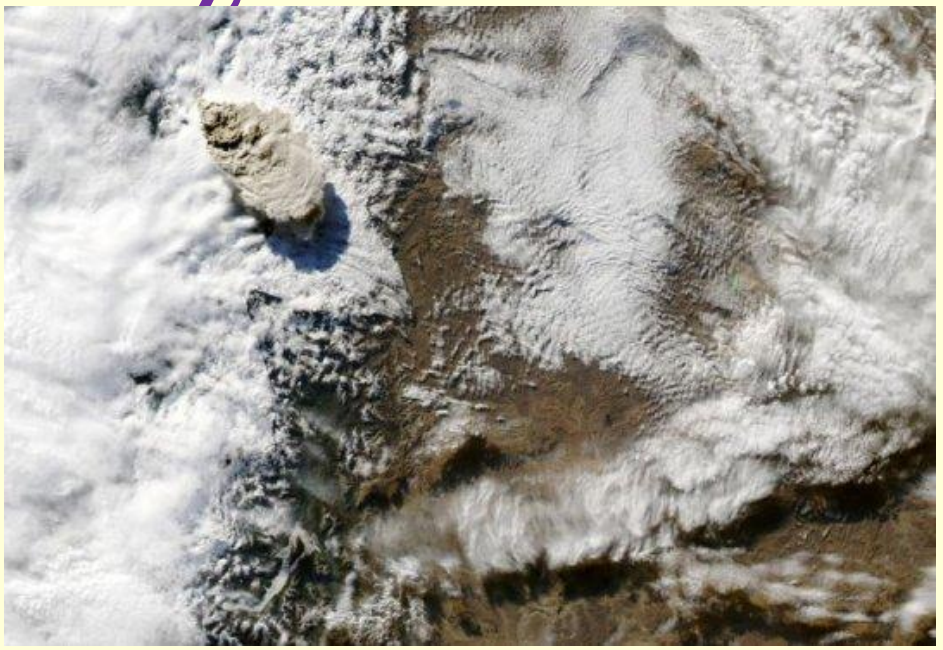


1. Experimental studies

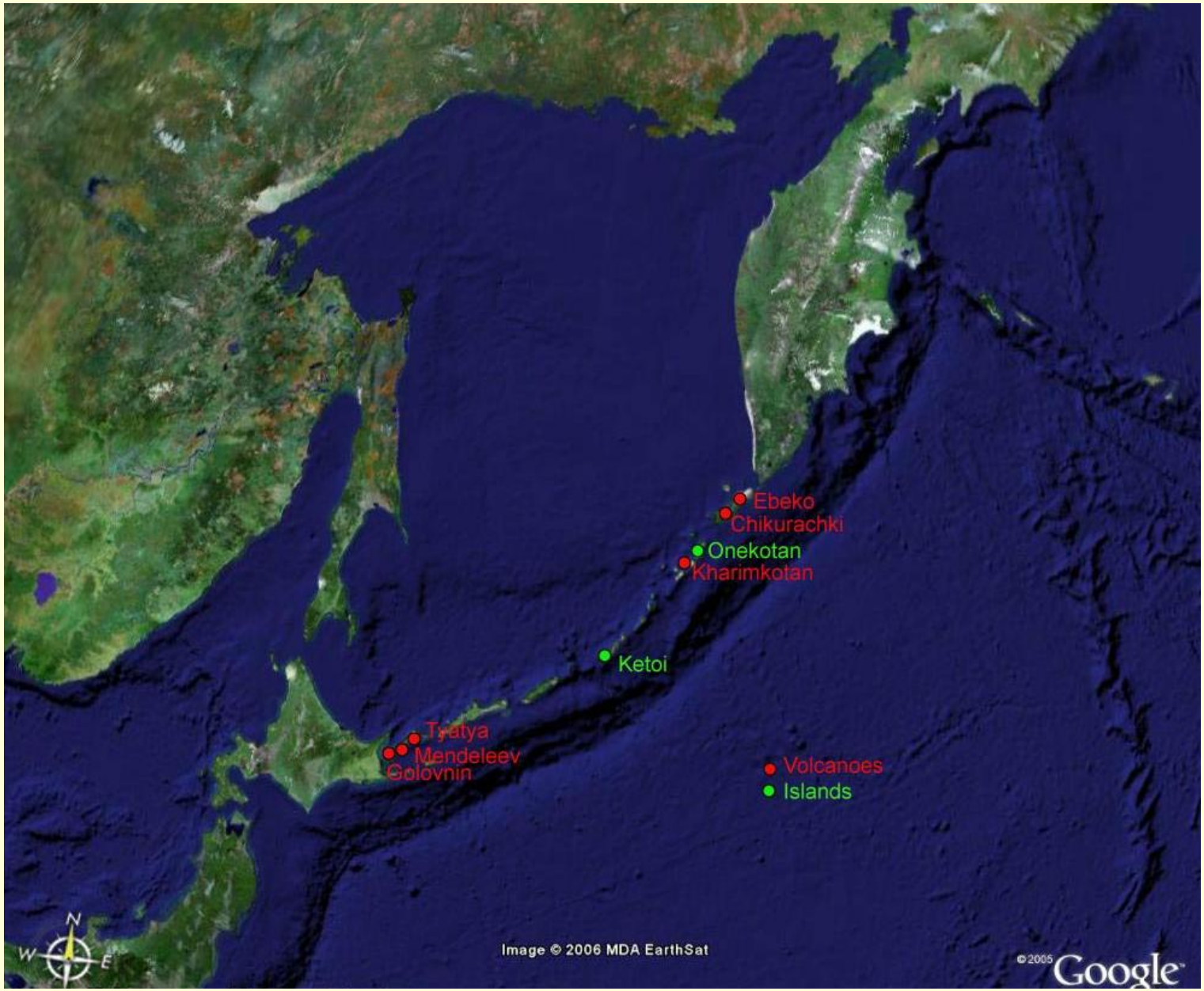


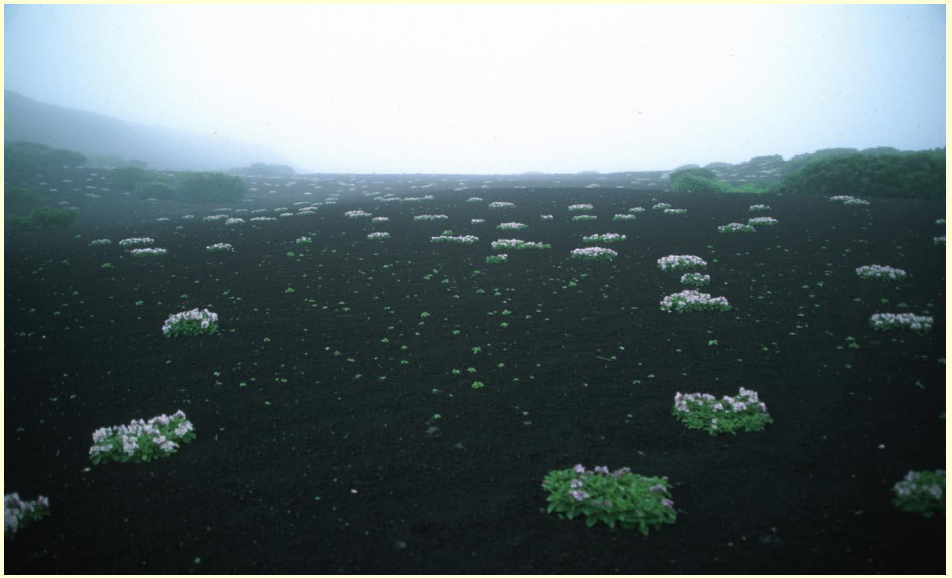
Fig. 1.

Paramushir Island.

Volcanoes.

Eruption of 2003, 2007

Tephra eruptions after 1853 and 1986



1. Problem statement

$$N(w) = \frac{a^{n+1}}{\Gamma(n+1)} w^n e^{-aw} \quad , \quad n \geq -1 \; , \quad a = \frac{n}{w_m} \qquad (1)$$

$$\sigma(x,y) = \int_0^\infty \int_0^\infty w \, q(x,y,0,w,t) \, N(w) dw dt \qquad (2)$$

$$\frac{\partial q}{\partial t} + u(z)\frac{\partial q}{\partial x} - w\frac{\partial q}{\partial z} = K_x \frac{\partial^2 q}{\partial x^2} + K_y \frac{\partial^2 q}{\partial y^2} \qquad (3)$$

$$q\big|_{t=0} = Q\delta(x)\delta(y)\delta(z-H) \qquad (4)$$

$$q\big|_{z>H} = 0 \quad ; \quad q \to 0 \ , \ |x| , |y| \to \infty , t \to \infty$$

$$K_x = \alpha \bar{U}_z^2 \frac{H-z}{w} \quad , \quad K_y = \beta \bar{U}_z^2 \frac{H-z}{w} \quad ,$$

$$\bar{U}(z) = \frac{1}{H-z}\int_z^H u(\xi)d\xi \qquad (5)$$

$$\sigma(x,0) = \frac{Q \cdot H \cdot \bar{U}_0 \cdot N(\frac{H\bar{U}_0}{x})}{\sqrt{2\pi\beta} \cdot x^3} \qquad (6)$$

$$P(x,\vec{\theta}) = \frac{\theta_1}{x^2} \int_{H_1}^{H_2} Q(h) \left(\frac{h}{x}\right)^{\theta_2} \exp\left(-\theta_3 \frac{h}{x}\right) dh \qquad (7)$$

$$\theta_1 = \frac{\left(a\bar{U}_0\right)^{n+1}}{\sqrt{2\pi\beta} \cdot \Gamma(n+1)} \quad , \quad \theta_2 = n+1 \quad , \quad \theta_3 = aU_0 \qquad (8)$$

2. Kinematic model estimation of ash fallout thickness

$$\frac{H}{w} = \frac{x}{U} \tag{9}$$

$$f(x,y) = \frac{1}{\sqrt{2\pi}\varphi_0 x} e^{-\frac{y^2}{2\varphi_0 x^2}} \tag{10}$$

$$N(w) = \frac{a^{n+1}}{\Gamma(n+1)} w^n e^{-aw}, \qquad n \geq -1, \qquad a = \frac{n}{w_m} \tag{11}$$

$$P\left(x, y, \vec{\theta}\right) = \theta_1 x^{\theta_2} \exp\left(-\frac{\theta_3}{x} - \frac{\theta_4 y^2}{x^2}\right), \qquad \textbf{(12)}$$

$$\theta_1 = \frac{T a^{n+1} \left(U H\right)^n}{\sqrt{2\pi}\varphi_0 \Gamma\left(n+1\right)}, \quad \theta_2 = -n-1, \quad \theta_3 = aUH, \quad \theta_4 = \frac{1}{2\varphi_0^2}$$

$$J\left(\vec{\theta}\right) = \sum_{j=1}^{M} \sigma_j^{-2} \left[r_j - P\left(x_j, y_j, \vec{\theta}\right)\right]^2 \to \min_{\vec{\theta} \in \Omega} \qquad \textbf{(13)}$$

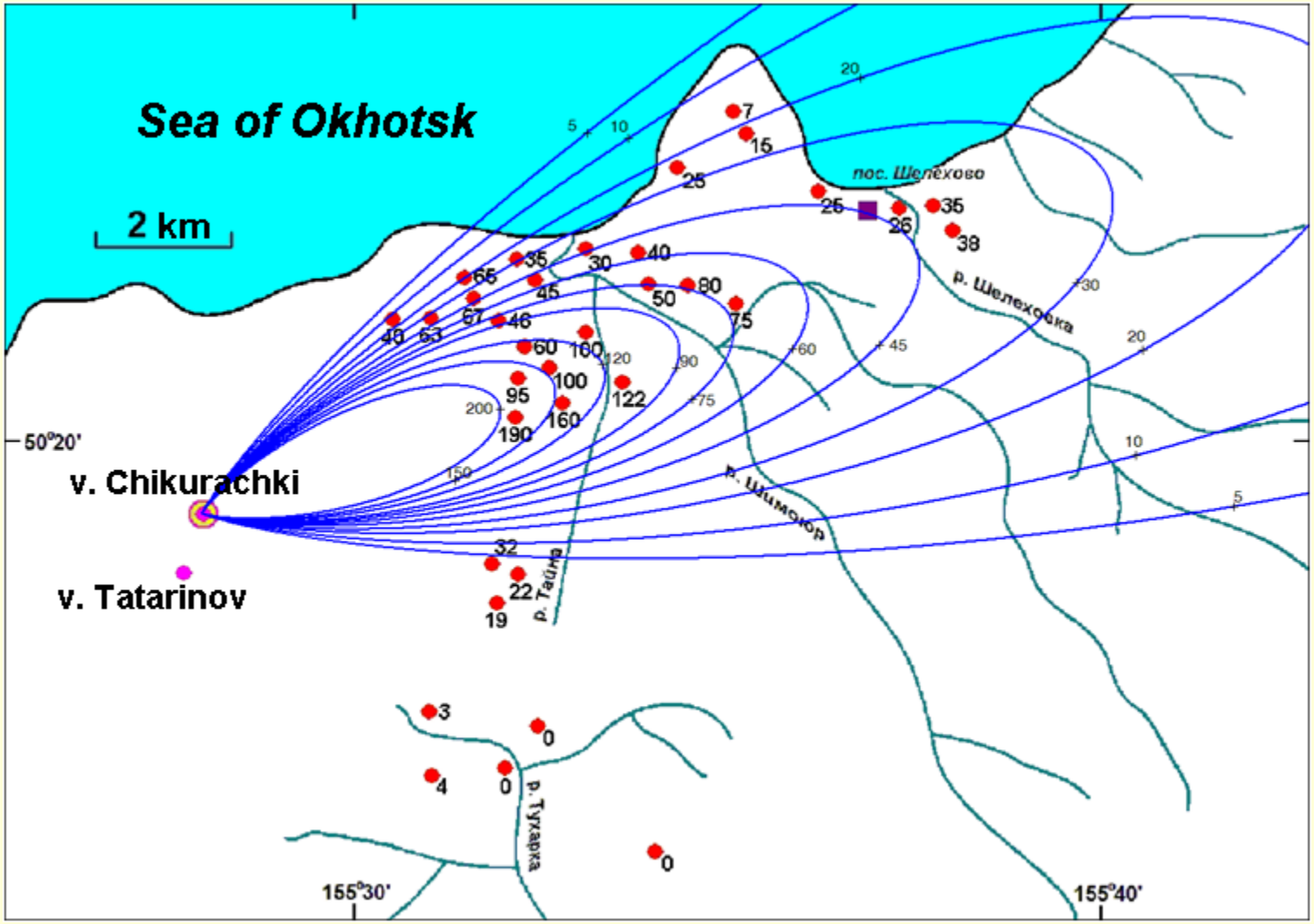


Fig. 2. The scheme of tephra sampling in the vicinity of the volcano Chikurachki, eruption **in 1853.** Recovered from model (12) field fallout. ● - position of sampling points.

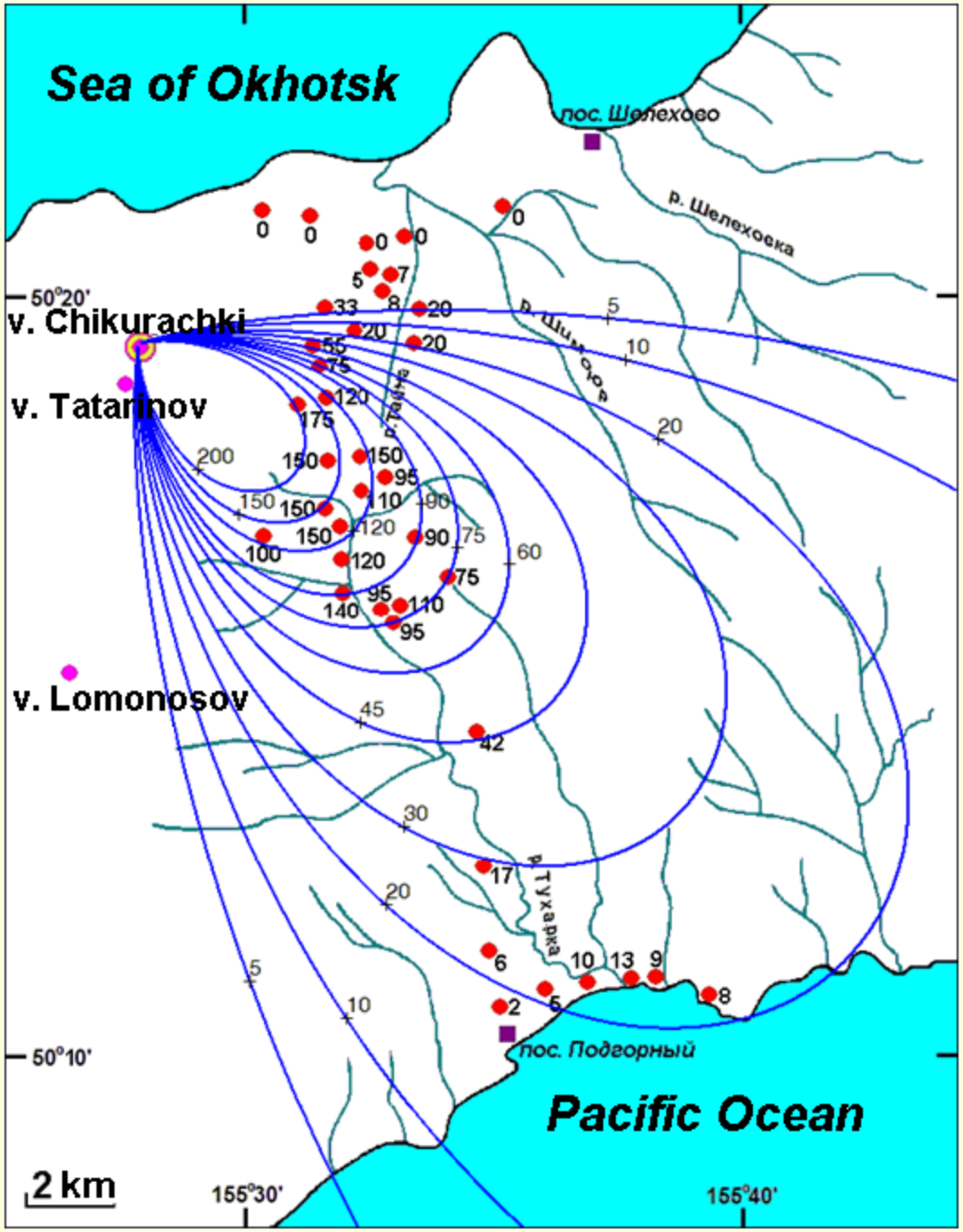


Fig. 3.
The scheme of tephra sampling in the vicinity of the volcano Chikurachki, eruption **in 1986.** Recovered from model (12) field fallout.

● - position of sampling points.

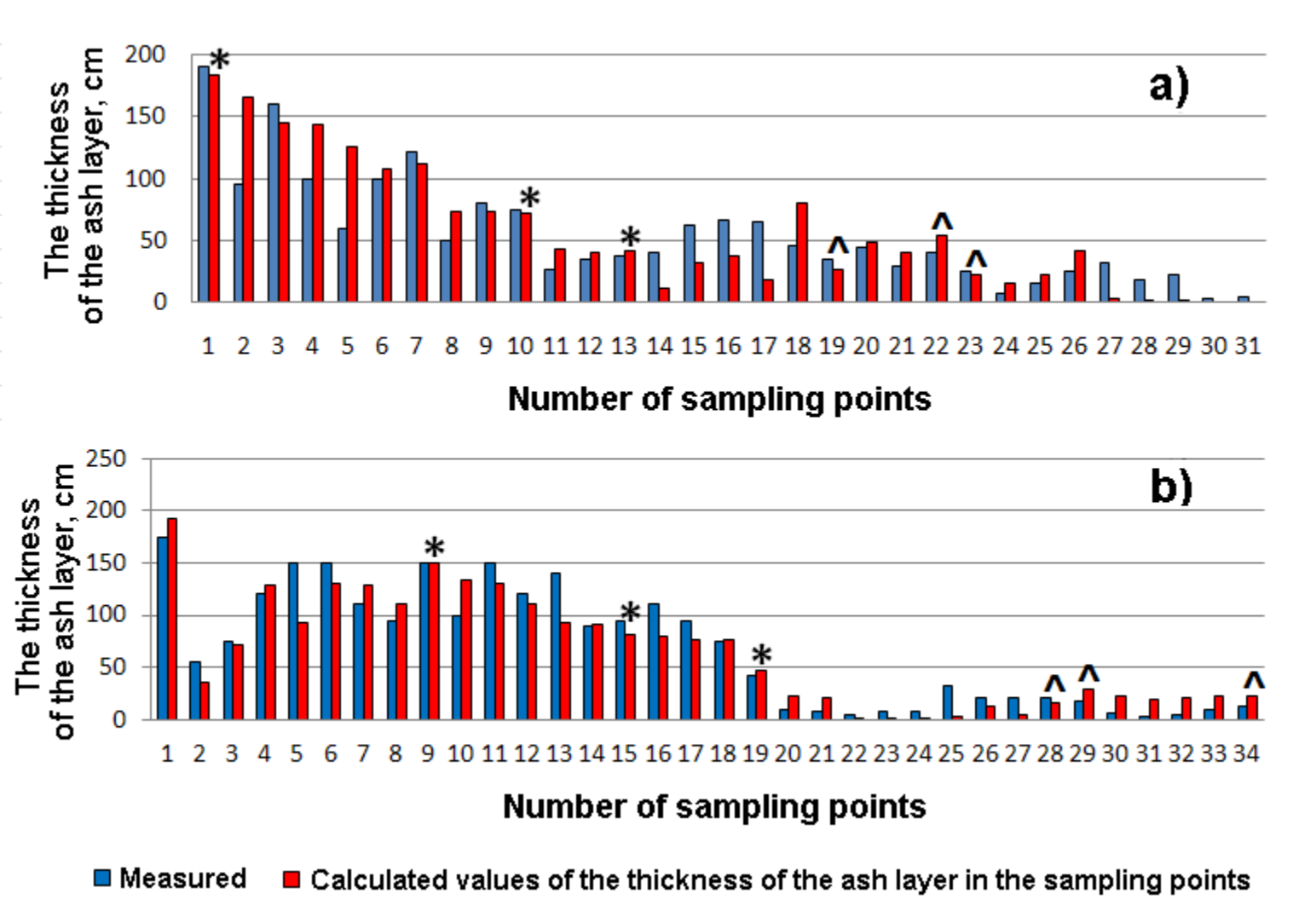


Fig. 4. Measured and numerical reconstruction of tephra fallout thickness at sampling points :
a) eruption of **1853**, **b)** eruption of **1986**

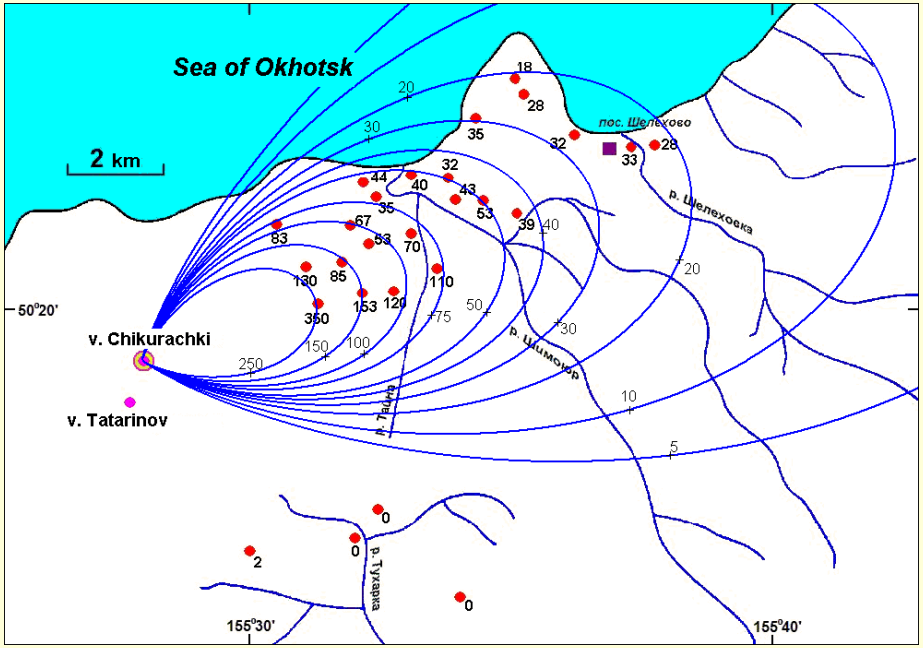
3. Model estimation of particle size of tephra

$$f(x,y)=\frac{1}{\sqrt{2\pi}\,\sigma(x)}\,e^{-\frac{y^2}{2\sigma^2(x)}}\quad,\quad \sigma^2\rightarrow k\,x^{2\omega}\qquad \omega\geq 0.5 \qquad \textbf{(14)}$$

$$w=c\,d^{\,2}\qquad\qquad d=\sqrt{\frac{w}{c}}=\sqrt{\frac{HU}{c\,x}} \qquad \textbf{(15)}$$

$$P\left(x,y,\vec{\theta}\right)=\theta_1\,x^{\theta_2}\exp\left(-\frac{\theta_3 y^2}{x^{2\omega}}\right) \qquad \textbf{(16)}$$

$$\theta_1=\sqrt{\frac{k\,HU}{2\pi c}}\quad,\quad \theta_2=-\omega-0.5\,,\quad \theta_3=\frac{1}{2k}$$



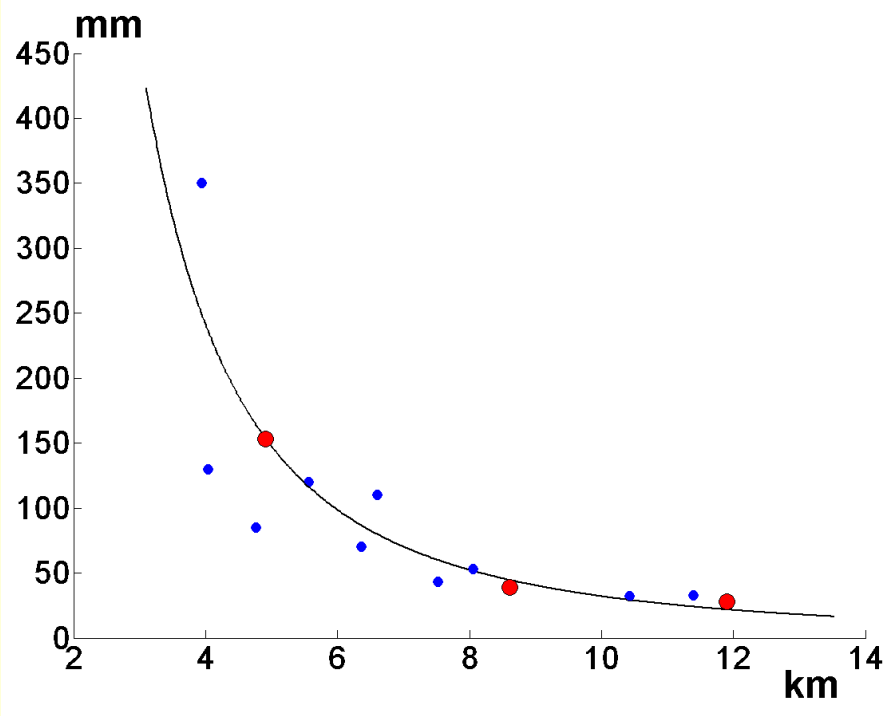
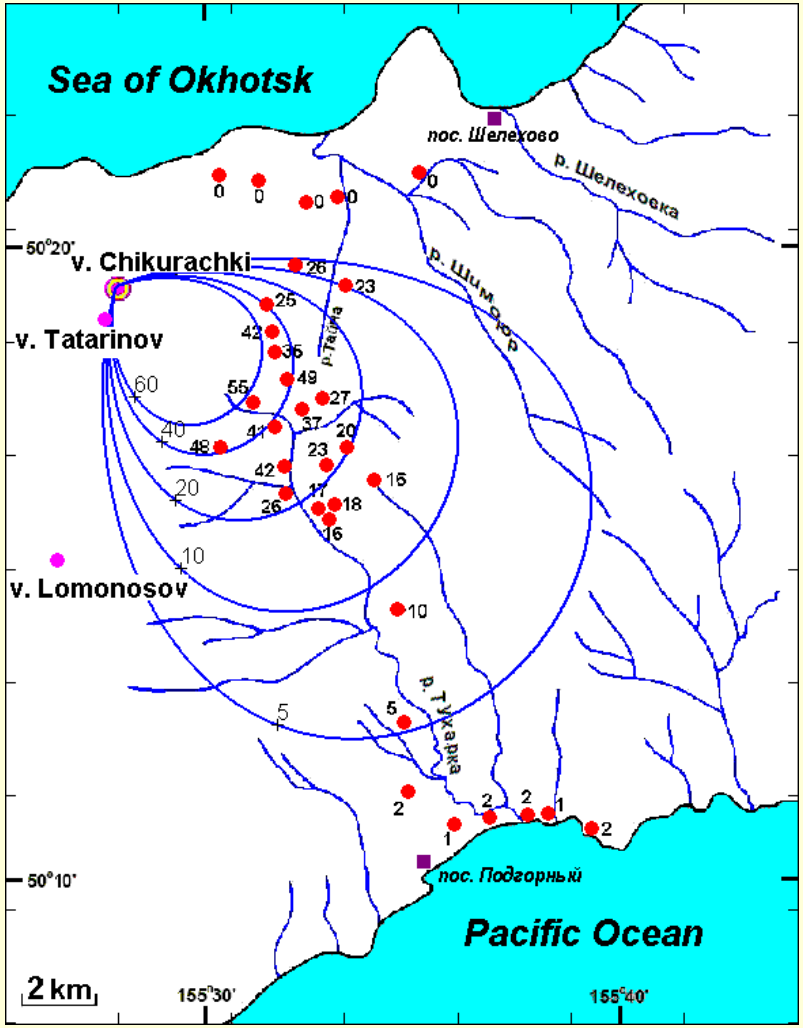


Fig. 5. Measured and numerically reconstructed particle size of tephra by volcanic axes ashfalls for **1853**.
● - support points, ● - control points of observation



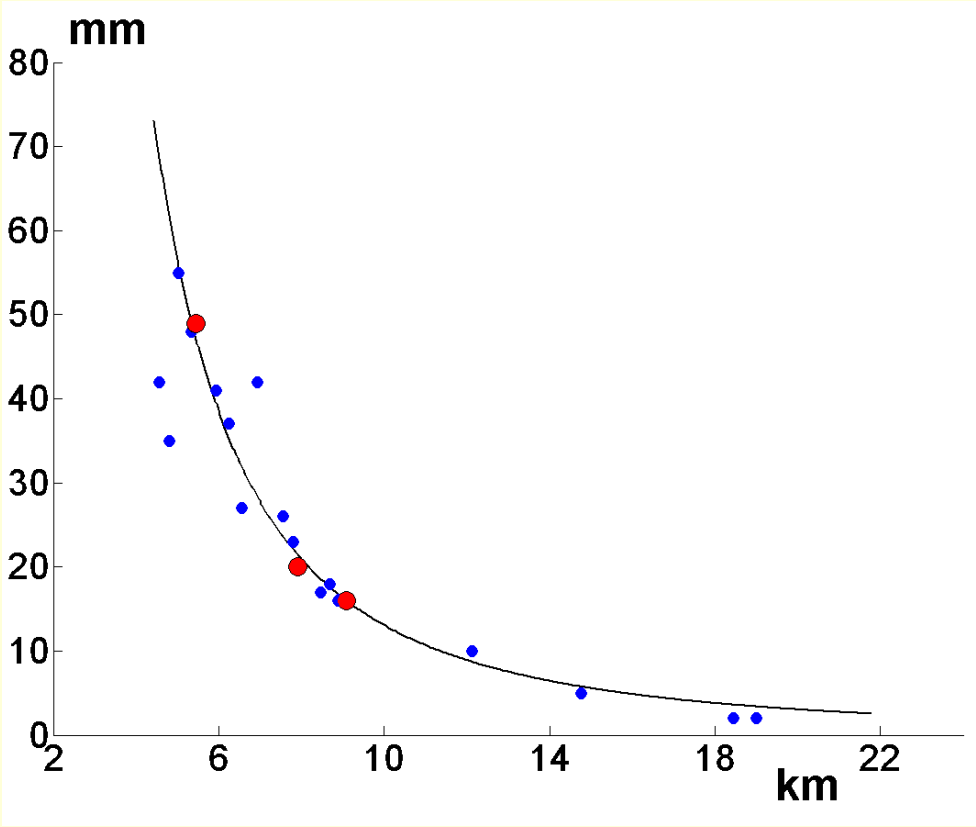


Fig. 6. Measured and numerically reconstructed particle size of tephra eruption for **1986**

Conclusion

- to reconstruct the fields of ash deposition thickness and particle size developed polydisperse models with small number parameters;

- was carried out testing of these models on field data in the vicinity of the volcano Chikurachki;

- to restore the field deposition can be used in a very limited number of measurement points, it creates certain advantages for the analysis of available data.

Thanks
for attention