Eddy diffusion in the atmosphere and at the ocean surface

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$k = 0.17\text{cm}^2/\text{s}$ diffusion coefficient of O_2 into N_2
diffusion equation, Adolf Fick, 1855,
after Charles Fourier, 1821.

$K \quad r \quad u$,eddy diffusion coefficient,

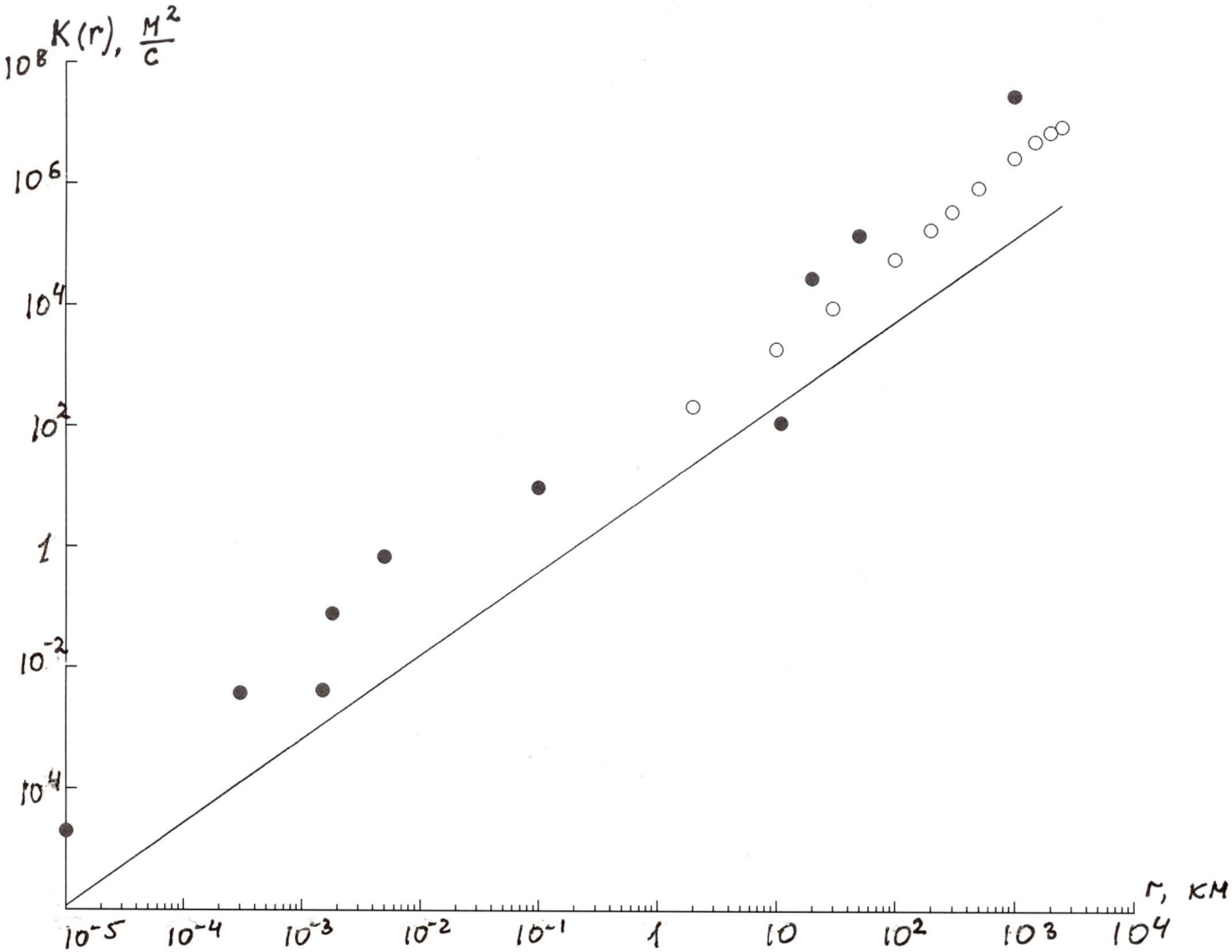
G.I. Taylor. 1915.

$K \sim r^{4/3}$ for the atmosphere,

L.F. Richardson 1926, 1929.

Compare the last two lines and get $u \sim r^{1/3}$!

Kolmogorov 1941



- Richardson 1926, 1929
- Golitsyn 2001

L.F. Richardson and Stommel, 6 January, 1948

(J. Meteorology 1948. V. 5. No. 5. 238 – 240)

parsnip white pieces ~ 1 inch relative distance l with time t

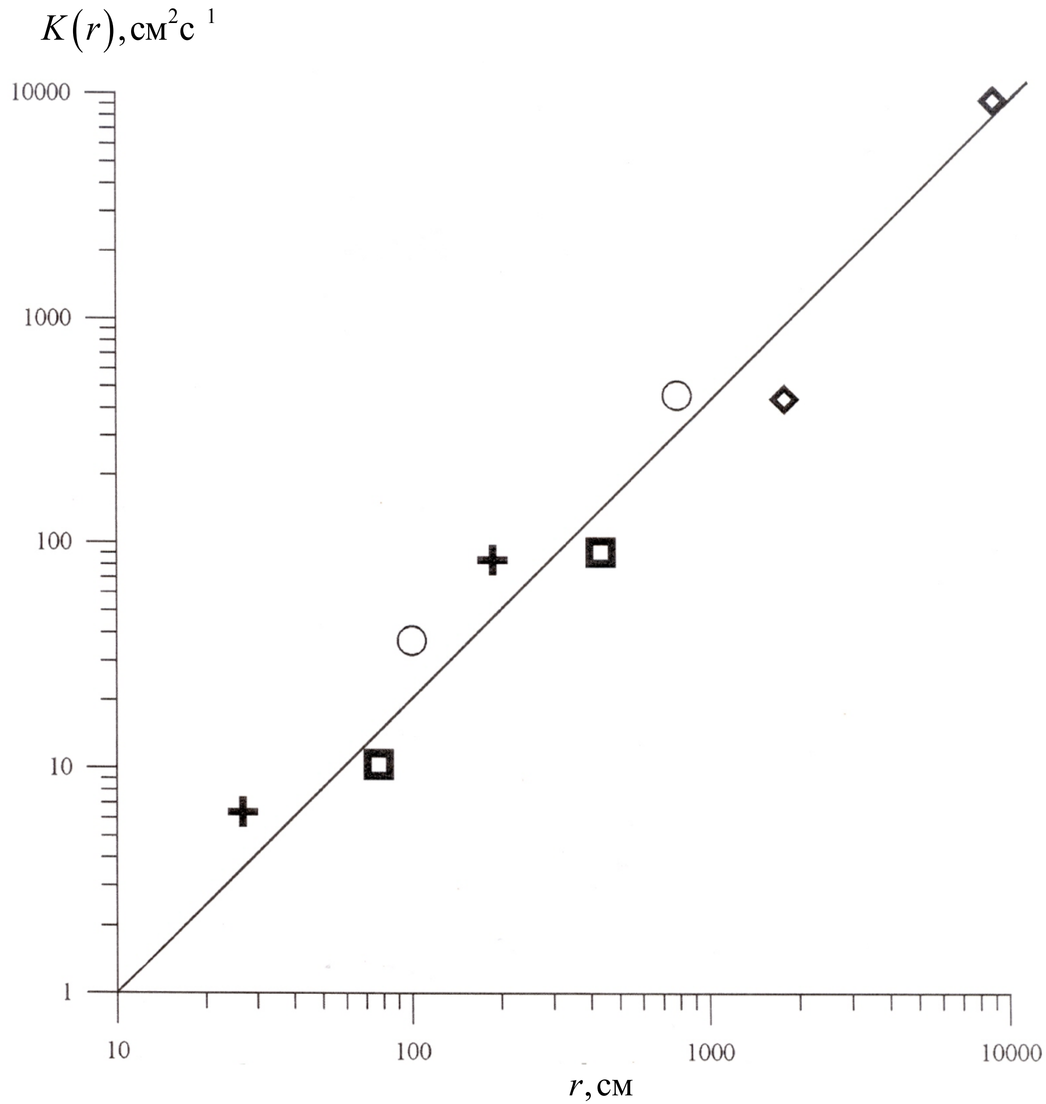
$$K(l) = \frac{1}{n} \quad _n \quad \frac{(l_0 \quad n^{1} \quad l)^2}{2t}$$

l_0 - initial distance between markers at $t = t_0$

~100 markers for about an hour, for $l \quad 3\mathrm{m}$, $t = 30$ sec.

$$K(l) \sim l^n, \quad n = 1.4$$ in ref. 1948

recalculation by myself $n = 1.32.$



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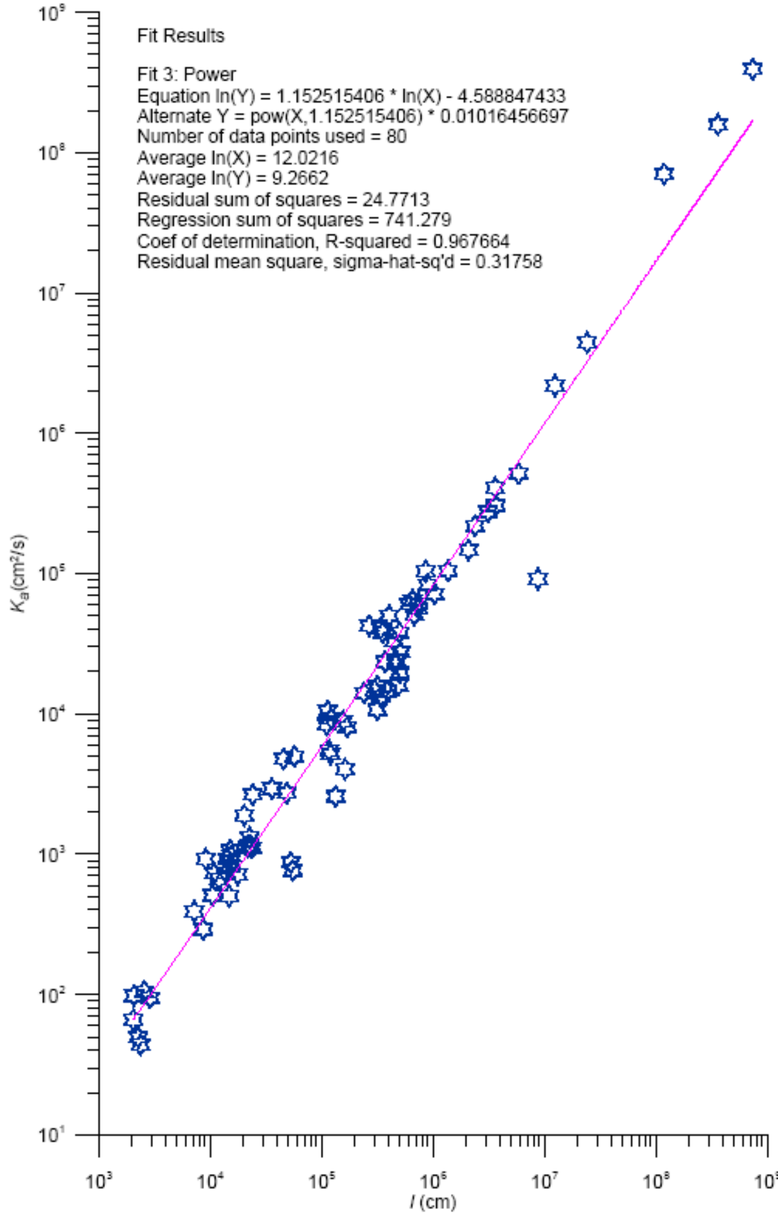
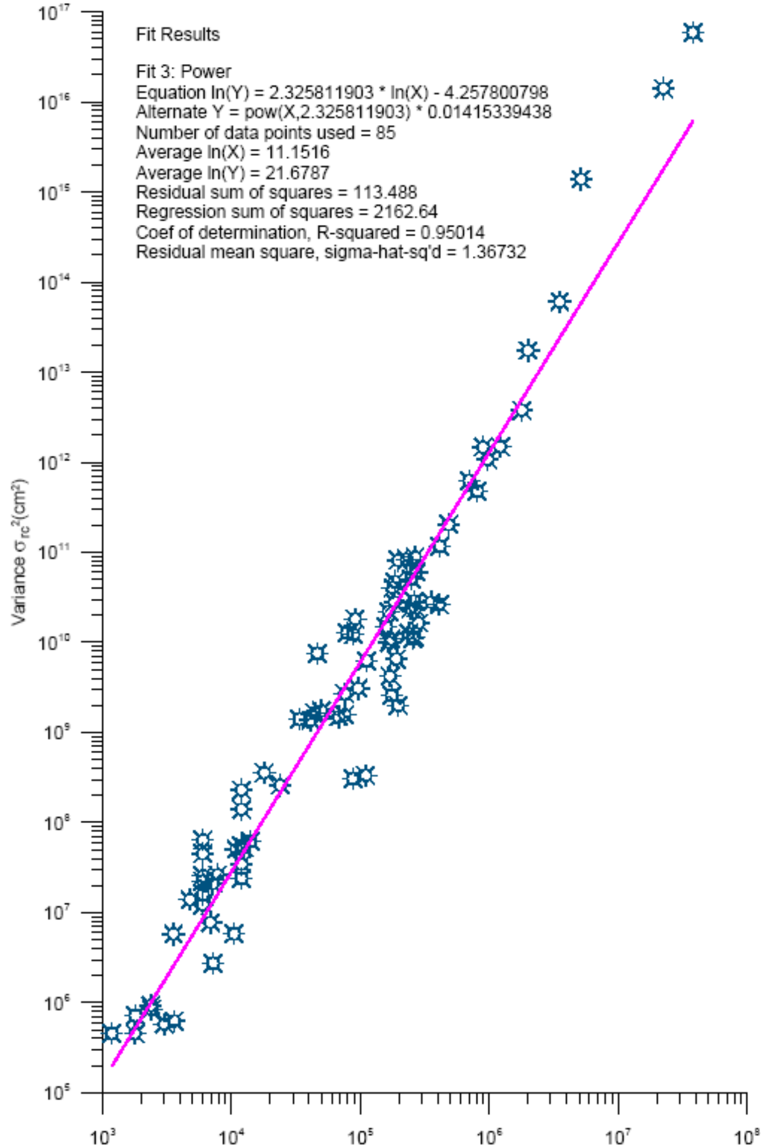
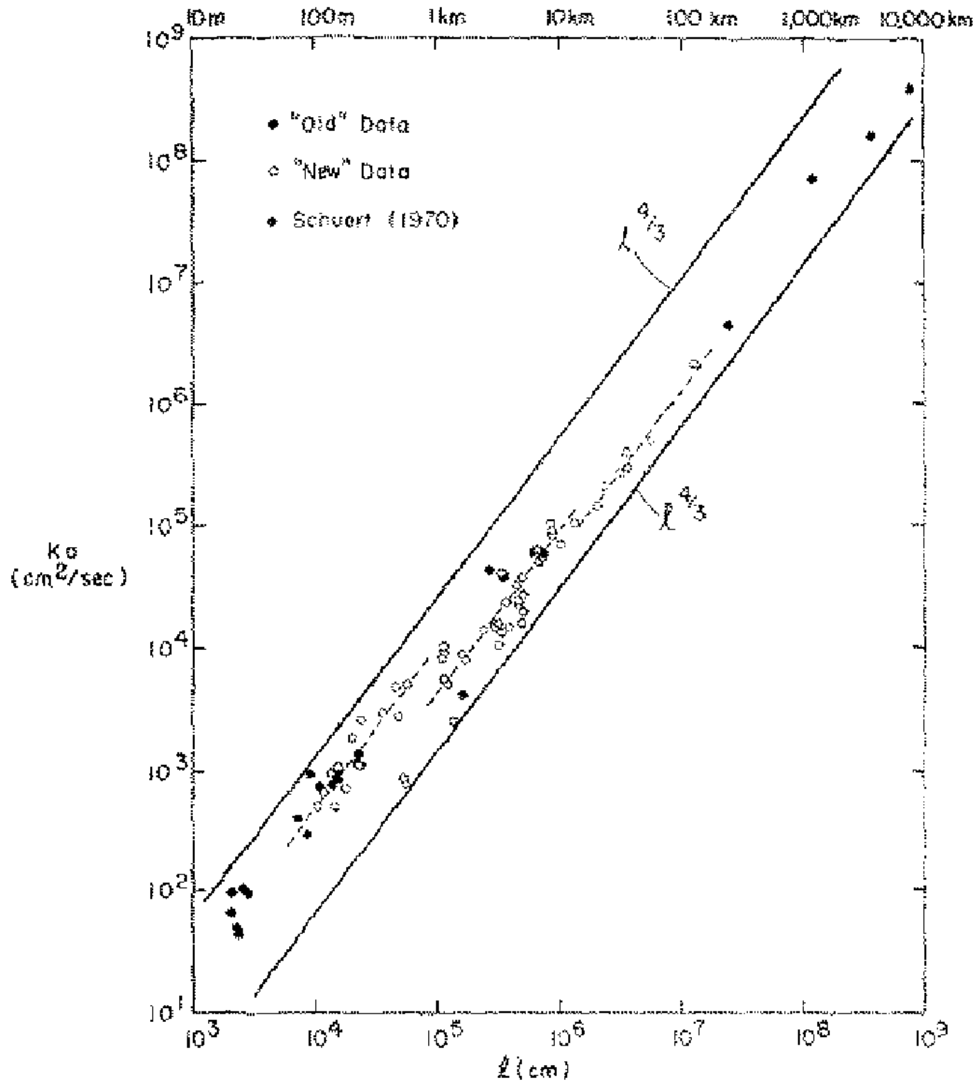


Diffusivity versus scale

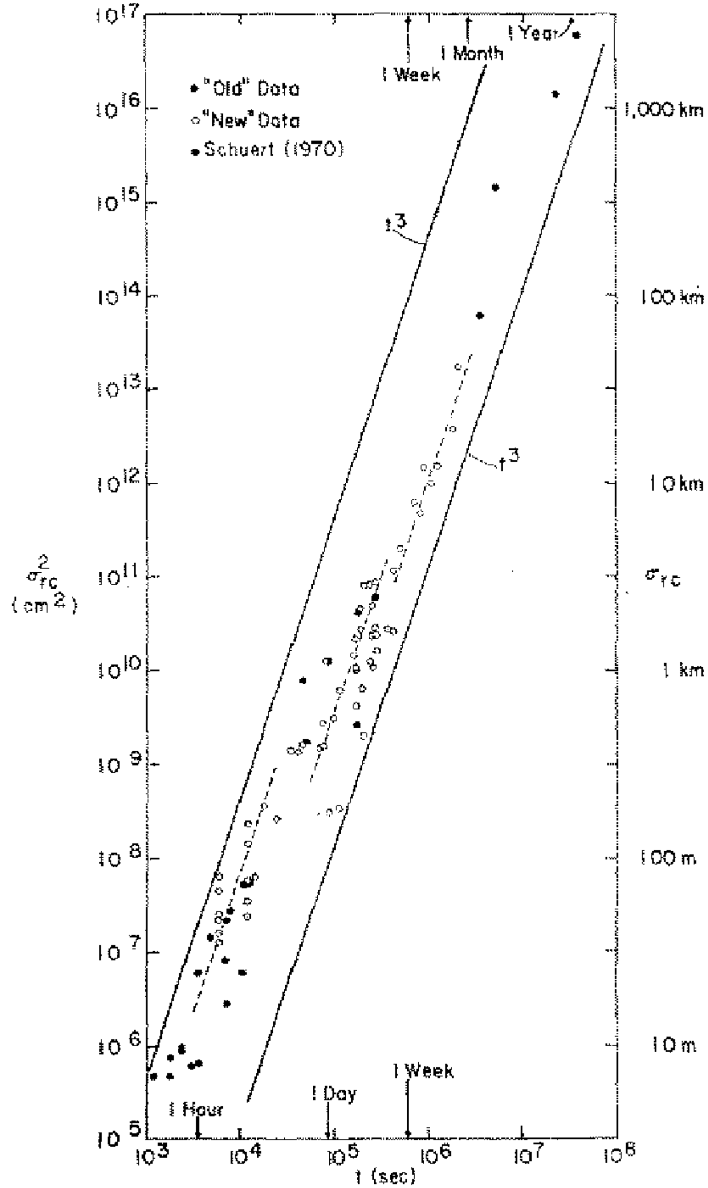


Variance versus time



10m
100m
1km
10km
100 km
1,000km
10,000km
"Old" Data
"New" Data
Schuert (1970)
Ka
(cm²/sec)
ℓ (cm)

3 c



10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
σ_{rc}^2 (cm^2)
"Old" Data
"New" Data
Schuert (1970)
1 Week
1 Month
1 Year
t^3
t^3
1,000 km
100 km
10 km
σ_{rc}
1 km
100 m
10 m
1 Hour
1 Day
1 Week
10^3
10^4
10^5
10^6
10^7
10^8
t (sec)

Taylor $K(r) \sim r \quad u, \quad , \quad u = \left\langle \; D_u(r) \;\; ^{1/2} \right\rangle = \left\{ \left\langle \; u(x+r) \quad u(x) \;\; ^{2} \right\rangle \right\}^{1/2}$

how to get $D_u(r)$, the structure function for velocity field in the sea surface waves?

We know the time elevation spectrum of the surface

$$E_z(\quad) \sim \quad ^{n}, \qquad\qquad n(\quad), \qquad\qquad = \frac{U_{10}}{c}$$ - the wave age

vertical velocity: $w = \dfrac{dz}{dt}, \qquad E_w(\quad) = \quad ^{2} E_z(\quad).$

Due to water incompressibility $E_u(\quad) \sim E_w(\quad) = \quad {}^2 E_z(\quad)$

dispersion relation: $\quad {}^2 = kg$

(Golitsyn 2007 general case for $n = 4$)

Probability transformation $E(\quad)d \quad = E(k)dk$

$$E_u(k) = c_{\mathrm{gr}} E_u(\quad) \sim c_{\mathrm{gr}} kg E_z(k(\quad))$$

$$D_u(r) = 2 \ _0 (1 \quad \cos r) E_u(k)\, dk$$

$$D_u(r) \sim r^{\frac{n \quad 3}{2}}; \qquad K(r) \sim r \ , \qquad = \frac{n+1}{4}$$

From the parabolic nature of the diffusion equation with scale variable diffusion coefficient $K(r) : \; r$

$$r^2 \quad S(t) \sim t^{\frac{2}{2}} = t \;\; , \qquad = \frac{8}{7 \quad n} \quad \text{at} \qquad = \frac{n+1}{4}$$

Okubo $= 1.15$ for $r \quad 1000\text{km}$

$= 2.34$ for 1 day $< t \quad 1$ month

3 locally

$n = 4$ Kitaigorodsky 1962, Zakharov 1966, Toba 1973

Energy transfer: $D_u(r) \sim r^{1/2}; \quad K(r) \sim r^{5/4}, \qquad = 1.25, \quad S(t) \sim t^{8/3}$

$n = \frac{13}{3} = 4 + \frac{1}{3};$ Hasselmann K., Hasselmann S. 1974

Spectral momentum transfer

$$D_u(r) \sim r^{2/3}, \qquad K(r) \sim r^{4/3}, \quad S(t) \sim t^{3},$$

$n = \frac{11}{3} = 4 \quad \frac{1}{3};$ Zakharov&Zaslavsky 1983:

Spectral action transfer: $D_u(r) \sim r^{1/3}, \; K(r) \sim r^{7/6}, \; S(t) \sim t^{12/5},$

$\sim t^{1}, \quad {}_{1} \quad 1/2,$ Golitsyn 2010

Gagnaire-Renou E., Benoit M., Badulin S.I. J. Fluid Mech. 2011 computations:

$n = \frac{13}{3}$ for $> 2,$ young waves, small fetch, $t \sim 1.5\text{hr}$

$= 4/3, \quad = 3$ Richardson & Stommel: $= 4/3!$

$n = 4$ for $1.2 < \quad < 2 \quad = 5/4, \quad = 8/3, \quad t = 2 \quad 4\text{hr}$

no reliable data

$n = 11/3$ for $0.83 < \quad 1.2$ old waves near saturation $t \quad 4\text{hr}$

$= 7/6, \quad = 2.4$ Okubo: $= 1.15, \quad = 2.34!$

Horace Lamb, 1895. Hydrodynamics

§ 349. Surface waves with viscosity. Linear approximation

$$u = \left(ikAe^{kz} + mCe^{mz} \right) e^{ikx+nt}, \qquad i = e^{i \quad /2}$$

$$w = \left(kAe^{kz} \quad ikCe^{mz} \right) e^{ikx+nt}, \qquad z < 0$$

Small parameter

$$_1 = \quad k^2 / \quad , \qquad k = 2 \quad / \quad ,$$

$$n = \quad 2 \quad k^2 \pm i \quad = \qquad \left(2 \quad _1 \text{ m} i \right)$$

$$\frac{C}{A} = \text{m} \frac{2 \quad k^2}{\quad} = \text{m} 2 \quad _1 << 1$$

$$m^2 = k^2 + n /$$

$$m^2 = \quad k^2 \pm i \quad /$$

The depth of the vorticity viscous layer $l \sim (\quad / \quad)^{1/2}$

$m^2 = \quad k^2 \pm i \quad /$ complex number with large imagenary part on the complex plane:

$$m^2 = \left(k^4 + \quad {}^2 / \quad {}^2\right)^{1/2} e^{i} \;, \qquad = \mathrm{arctg} \quad \text{m} \frac{}{k^2} \quad = \mathrm{arctg}\left(\pm 1/ \quad\right) = \frac{}{2} \quad 2 \quad ,$$

$$m = \left(k^4 + \quad {}^2 / \quad {}^2\right)^{1/4} e^{i \quad /2} \quad \frac{k}{{}_1^{\;1/2}} e^{\text{m}i(\quad /4 \quad)}.$$

For typical ocean waves (Golitsyn, 2010) $h = 2.7$ m, $=40$ m, $=1.25$ c^{-1},

$$_1 = \frac{k^2}{} = \frac{4 \quad ^2}{\quad ^2} = 2.4 \quad 10 \quad ^8, \qquad {}_1^{1/2} = 1.56 \quad 10 \quad ^4$$

At $z = 0:$

$$u = \ (ikA + mC)e \ = \ kA \ \ i + \frac{m}{k}\frac{c}{A} \ \ e \ = \ kA \ \ i + (2 \ \ )^{1/2} \ \ e$$

$$w = \ (kA \ \ ikC)e \ , \qquad = ikx + nt = i(kx \pm \ \ t) \ \ 2 \ \ k^2 t = i \ \ _1 \ \ 2 \ \ k^2 t$$

$$u = \ 1 + (2 \ \ )^{1/2} \ \ ^{1/2} \ e^2 \ , \qquad = \text{arctg}\frac{1}{(2 \ \ )^{1/2}} = 90^\circ \qquad , \qquad = 0.69 \ = 41$$

$$w = \ kA(1 \ \ ic / A)e \ = \ kA(1 \ \ 2i \ \ )e^i \ = \ kA\left(1 + 4 \ \ ^2\right)^{1/3} e^{i \ \ _1},$$

$$_1 = \text{arctg}2 \ \ = 90^\circ \qquad _1, \qquad _1 = 0.00\ 4$$

permanent addition to the phase of horizontal wave of order one minute!

Comparing to the phase of vertical component!

Slow horizontal diffusion motion on x!

Mean momentum in time:

$$\langle uw\rangle = \left\langle k^2 A^2 \sin\left(\quad _1 + 2 \quad ^{1/2}\right)\cos \quad \right\rangle = k^2 A^2 \left\langle \left(\sin \quad \cos 2 \, _1^{1/2} \pm \cos \quad \sin 2 \, _1^{1/2}\right)\cos \quad \right\rangle$$

$$k^2 A^2 \, _1^{1/2} = \quad ^2 h^2 \, _1^{1/2} = \quad ^2 h^2 \, _1^{1/2}.$$

For the mean waves this corresponds to 1 cm/s

For the mean wind of 8 m/s the Stokes drift:

$u_{\ a} = 29$ cm/s and $u_{\text{drift}} \quad 0.5 u_{\ a} = 15$ cm/s, J. Wu 1975.

But the non-linear Stokes drift is over the whole region of wind action moving as a whole the pollution spot, synoptic scale 1000 km.

$$r^2 = 0.0108 t^{2.34}, \quad r = 0.1 t^{1.17} \ \text{ slightly overballistic motion of a spot boundary}$$

$$r^2 = \quad t^3 \quad \text{for isotropic turbulence } r \sim t^{1.5}, \quad \text{when } K \sim \quad ^{1/3} r^{4/3}$$

Due to G.I. Taylor (1915) the diffusion coefficient

$$K(r) = \langle a_2 r \;\; u(r) \rangle = a_2 r \;\; D_u(r) \;\; {}^{1/2}, \quad a_2 \quad \text{number.}$$

For $n = 11/3$:

$$K(r) = a_2 r \;\; {}_1^{1/4} \;\; 3.52 \;\; \frac{8}{3} h^2 \;\; {}_p^{8/3} g \;\; {}^{1/3} r^{1/3} \;\; {}^{1/2} \sim r^{7/6}$$

Okubo: $K(r) = 0.0103 r^{1.15}$

We recalculated from the tables by Okubo $= 1.15 \pm 0.05$

Comparing the theory with experiment for the annual mean wave field for the

World Ocean (Golitsyn, 2010) we find ${}_1^{1/2} = 1.56 \;\; 10 \;\; {}^4$ and $a_2 = 2.3 \;\; 10 \;\; {}^3$.

Okubo for the area of tracer spot:

$$S(t) = 0.0108 t^{2.34} = r^2, \quad \text{in cm}^2.$$

Diffusion eguation

$$\frac{S}{t} = \frac{}{r} K(r) \frac{S}{r}, \quad K(r) = br \quad ,$$

where $[b] = L^2 \quad T^{\ 1}$- constant over t and r. Introduce $= bt$, $[\quad] = L^{2-}$.
Dimensional analysis gives for $[S] = r^2$:

$$S = b_1 \quad , \qquad = \frac{2}{2}$$

With $= 7/6$ we get $= 12/5 = 2.4$, while reanalyzing the Okubo tables =2.33+0.10. Our result gives for $b = 460$. If we take 1971 value =1.15 then $= 2(2 \qquad)^{-1}$ =2.35. Both values =1.15 and $= 2.34$

Okubo obtained by eye!!

From Stommel $K(r) = 4.6\ \ 10\ ^{2} r^{4/3}$, our estimate

for $\ \ = \frac{2}{2-} = 3$, i.e. $S = r^2 \sim t^3$,

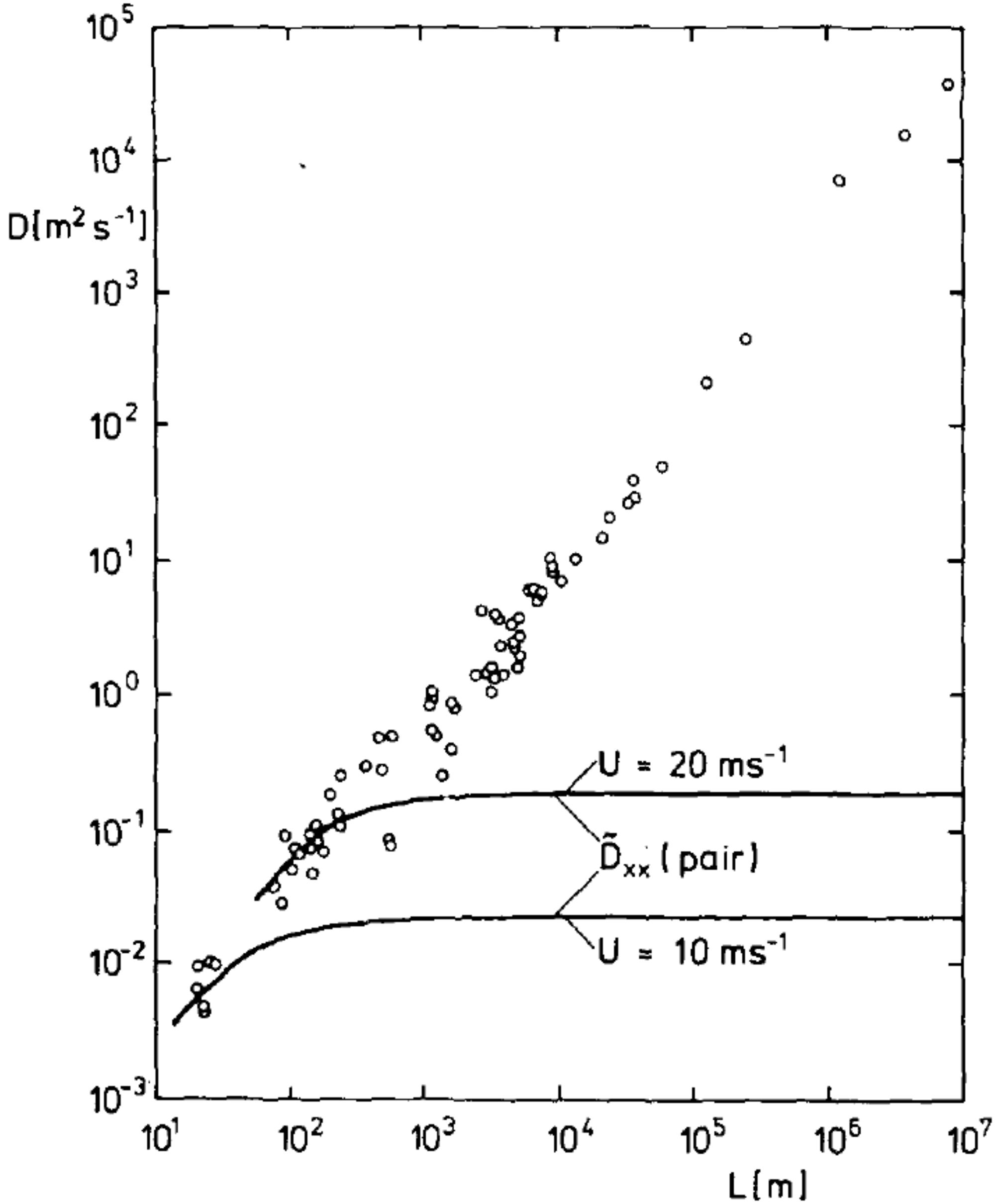
as for the case of locally homogeneous and isotropic

turbulence, Batchelor, 1950.

This rheory based on linear approximation (Lamb, 1895)

is self-consistent but determines constants, which depend on wind and fetch, from observations.

The limits of observational results corresponds to theory and numerics of Gagnaire-Renou E. Benoit M., Badulin S.I. 2011, JFM, 669, 178-213.



10^5
10^4
$D [m^2 s^{-1}]$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$U = 20\ ms^{-1}$
$\tilde{D}_{xx}$ (pair)
$U = 10\ ms^{-1}$
10^1
10^2
10^3
10^4
10^5
10^6
10^7
L [m]

K. Herterich&K. Hasselmann

The horizontal diffusion of tracers by surface waves JPO 1982. V. 12, 704 – 711.

Random fluctuations of the local Stokes-drift current, Pirson_Moscowitz can be explained wave spectrum diffusion coefficients for single particle, particle pairs and continous traces spot small scales up to hundreds m .

Thanks for your attention!