Direct and Inverse Relations Having an Impact on Formation of Hydrodynamic Regime and Quality of the Atmosphere

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Functionals for studies of direct and inverse relations

Goal functionals

$$\Phi_k(\boldsymbol{\varphi}) = \int_{D_t} F_k(\boldsymbol{\varphi})\chi_k(\mathbf{x},t)dDdt \equiv \left(F_k, \chi_k\right), \quad k = 1,...,K$$

F_k are evaluated functions (differentiable in generilized sense, bounded, satisfying the Lipschitz's conditions),

$\chi_k dDdt$ are Radon’s or Dirac’s measures on D_t, $\chi_k \in \Im^*(D_t)$.

Quality functionals for data assimilation

$$\Phi_k(\boldsymbol{\varphi}) = \int_{D_t} (\Psi - H(\boldsymbol{\varphi}))_m^T M(\Psi - H(\boldsymbol{\varphi}))_m \chi_m(\mathbf{x},t)dDdt,$$

$\chi_m dDdt$ Radon’s or Dirac’s measures $\left\{\boldsymbol{\Psi}(D_t^m) \Rightarrow \boldsymbol{\Psi}(D_t)\right\}$

Functionals for studies of direct and inverse relations

"Measurement" functionals for receptors

$$\Phi(\varphi) = \sum_{k=1}^{K} \int_{D_t} \left[H(\varphi) \right]_{rk} \delta(\mathbf{x} - \mathbf{x}_{rk}) dD\,dt,$$

$\mathbf{x}_{rk} \in D_t^r$ receptors locations,

$\delta(\mathbf{x} - \mathbf{x}_{rk}) dDdt$ Dirac's measure for $D_t^r \Rightarrow D_t$

Functionals for assessment of distributed restrictions

$\varphi(\mathbf{x}, t) \le N, \quad \vartheta_k(\varphi(\mathbf{x}, t) \le 0$ distributive constraints

$$\mathbf{\Phi}_k(\varphi) = \int_{D_t} (\vartheta_k(\varphi) + |\vartheta_k(\varphi)|) \chi_k(\mathbf{x}, t) dDdt = 0$$

$\chi_k dDdt$ are Radon's or Dirac's measures for constraints on $D_{tc} \subset D_t$,
$\chi_k \in \mathfrak{I}^*(D_t)$.

Some elements of optimal forecasting and design

The main sensitivity relations

$$\delta\Phi_k^h(\varphi) \equiv (\mathbf{\Gamma}_k, \delta\,\mathrm{Y}) \equiv \frac{\partial}{\partial\alpha} I^h(\varphi, \mathrm{Y}+\alpha\;\;\delta\,\mathrm{Y},\;\; \varphi_k^*)\Big|_{\alpha=0}$$

Algorithm for calculation of sensitivity functions

$$\mathbf{\Gamma}_k = \frac{\partial}{\partial\delta\,\mathrm{Y}}\left(\frac{\partial}{\partial\alpha} I^h(\varphi, \mathrm{Y}+\alpha\;\;\delta\,\mathrm{Y}, \varphi_k^*)\Big|_{\alpha=0}\right)$$

$\mathbf{\Gamma}_k = \{\Gamma_{ki}\}$ are the sensitivity functions

$\delta\,\mathbf{Y} = \{\delta Y_i\}$ are the parameter variations

$$k = \overline{1,K},\;\; i = \overline{1,N}$$

The feed-back relations

$$\frac{dY_\alpha}{dt} = -\eta_\alpha \mathbf{\Gamma}_{k\alpha}, \quad \alpha = \overline{1, N_\alpha}, \quad N_\alpha \le N, \;\; t_{j-1} \le t \le t_j$$

$\mathbf{\Gamma}_{k\boldsymbol{\alpha}}$ are the sensitivity functions

$\mathrm{Y}_{\boldsymbol{\alpha}}$ are the parameters to be refined

η_α are the descend parameters for mininization of the quality functional

Real time equations of feed-back relations with *a priori* information

$$\Phi_k(\boldsymbol{\varphi}, \mathbf{Y}) = \Phi_{ks}(\boldsymbol{\varphi}) + \Phi_{kp}(\mathbf{Y})$$

$$\Phi_{kp}(\mathbf{Y}) = 0.5\int_{D_t}\left\{\sum_{i=1}^{N}\left(\gamma_1\Gamma_{ip}^{(1)}\left|\mathrm{grad}\left(Y_i - \tilde{Y}_i^0\right)\right|^2 + \gamma_2\Gamma_{ip}^{(2)}\left(Y_i - \tilde{Y}_i^0\right)^2\right)\right\}dDdt$$

$$\frac{\partial Y_i}{\partial t} = -\kappa\frac{\partial\Phi_k(\boldsymbol{\varphi}, \mathbf{Y})}{\partial Y_i}, \quad i = \overline{1, N}; \quad \kappa \cong \Phi_k(\boldsymbol{\varphi}, \mathbf{Y}) / \left(\frac{\partial\Phi_k}{\partial\mathbf{Y}}, \frac{\partial\Phi_k}{\partial\mathbf{Y}}\right)$$

$$\frac{\partial Y_i}{\partial t} = -\kappa\left\{\frac{\partial I^h(\boldsymbol{\varphi}, \mathbf{Y}, \boldsymbol{\varphi}^*)}{\partial Y_i} - \gamma_1\,\mathrm{div}\,\Gamma_{ip}^{(1)}\,\mathrm{grad}\left(Y_i - \tilde{Y}_i^0\right) + \gamma_2\Gamma_{ip}^{(2)}\left(Y_i - \tilde{Y}_i^0\right)\right\}$$

$\tilde{Y}_i^0$ - a priori parameter values

γ_1, γ_2 - weight coefficients

$\Gamma_{ip}^{(\alpha)}$ - matrices of scale coefficients and weights

Algorithms of fast data assimilation on the base of variational principle, splitting schemes and uncertainty assessment

- Theoretical base: minimization of the functional of sum measure of uncertainty of the processes' models and the observations' models

- Decomposition of domains and functionals
- Local adjoint problems in the "windows" of assimilation with uncertainty assessment

Real time sequential data assimilation with adjoint problems

Algorithm 1. Uncertainty and adjoint functions taken explicitly

$$\left(E+\Delta t\Lambda_{sj}\right)\varphi^{j}-\left(f_s^{\,j}+(M_{2j}^{-1}/\alpha_{2j})\varphi^{*j}\right)\Delta t-\varphi^{j-1/n}=0$$

$$\left(E+\Delta t\Lambda_{sj}^{*}\right)\varphi^{*j}=\alpha_{1j}\Delta t\left[\frac{\partial H}{\partial\varphi}\right]^{T}M_{1j}(\Psi^{j}-H^{j}(\varphi^{j}))$$

Algorithm 2 . Uncertainty and adjoint functions included to general scheme

$$\alpha_2\left(\boldsymbol{E}+\Delta\boldsymbol{t}\;\;\Lambda_{sj}^{*}\right)\hat{\boldsymbol{M}}_2\left(\left(\boldsymbol{E}+\Delta\boldsymbol{t}\;\;\Lambda_{sj}\right)\varphi^{j}-\Delta\boldsymbol{t}\mathbf{f}^{j}-\varphi^{j-1/n}\right)$$

$$+\alpha_1\Delta\boldsymbol{t}\left[\frac{\partial\boldsymbol{H}}{\partial\varphi}\right]^{T}\hat{\boldsymbol{M}}_1\left(\left(\boldsymbol{H}\varphi\right)_m^{j}-\Psi_m^{j}\right)=0$$

Λ_{sj} - Operator of splitting stage, s- stage number

Additive splitting schemes → Parallel algorithms

Variational 4D data assimilation

- Decomposition of domains and functionals

$$D_t^h = \bigcup_{j=1}^{J-1} D_{tj}^h, \quad D_{tj}^h = D^h \times \left[t_{j-1}, t_j\right], \quad \tilde{\Phi}^h(\boldsymbol{\varphi}, \boldsymbol{\varphi}^* \mathbf{Y}, \Psi) = \sum_{j=1}^{J-1} \sum_{s=1}^{p} \tilde{\Phi}_{js}$$

•Subgrid data structure

$$\left\{\boldsymbol{\varphi}_s^j, \boldsymbol{\varphi}_s^{*j}, \mathbf{r}_s^j, \ \ s = \overline{1,p}\right\} = \bigcup_{s=1}^{p} Q_s^h\left(D_t^h\right) \subset Q^h\left(D_t^h\right)$$

Data transfer to subgrid structure

$$\left\{\boldsymbol{\varphi}_s^{j-1} \in Q^h\left(D_t^h\right)\right\} \Rightarrow \bigcup_{s=1}^{p} \left\{\boldsymbol{\varphi}_s^{j-1} \in Q_s^h\left(D_t^h\right)\right\}, \boldsymbol{\varphi}_s^{j-1} = \boldsymbol{\varphi}^{j-1}, \ s = \overline{1,p}, t = t_{j-1}$$

Parallel algorithm

- Solution of assimilation problem on splitting steps in parallel

$$\Lambda_s^j \varphi_s^j - f_s^j - r_s^j = 0$$

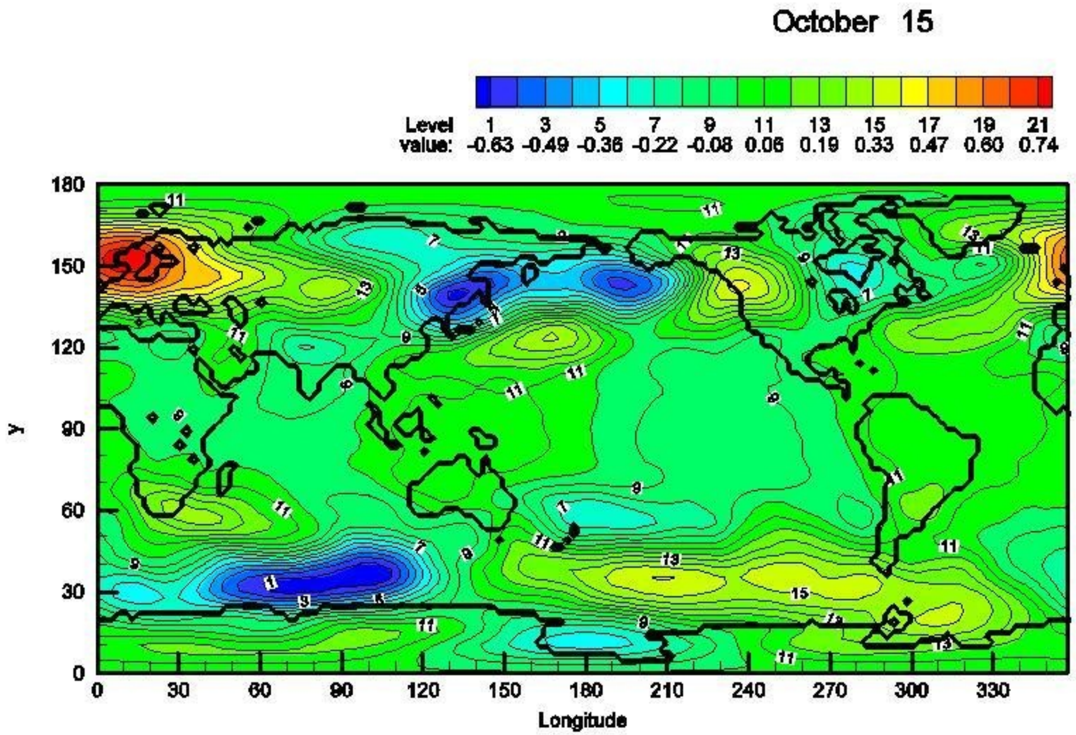
$$\Lambda_s^{*j} \varphi_s^{*j} = \left[\frac{\partial \Phi_{ks}(\varphi)}{\partial \varphi} + \left(\frac{\partial \left[H(\varphi) \right]_m}{\partial \varphi} \right)^T M_{1j} (\Psi_m - \left[H(\varphi) \right]_m) \right]_s^{j-1}$$

$$\varphi_s^{*j+1} = 0, \quad r_s^j = \left(M_{2s}^j \right)^{-1} \varphi_s^{*j}, \quad t_{j-1} \le t \le t_j, \quad s = \overline{1, p}$$

- Data transfer from subgrid structure onto basic structure

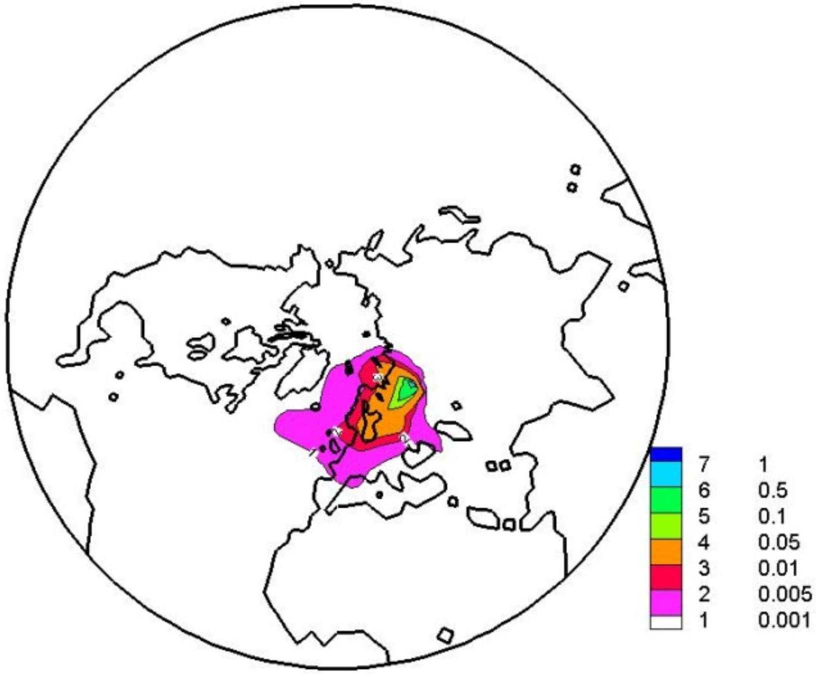
$$\bigcup_{s=1}^{p} \left\{ \boldsymbol{\varphi}_s^j \in Q_s^h \left(D_t^h \right) \right\} \Rightarrow \left\{ \boldsymbol{\varphi}_s^j \in Q^h \left(D_t^h \right) \right\}, \quad \boldsymbol{\varphi}^j = \frac{1}{p} \sum_{s=1}^{p} \boldsymbol{\varphi}_s^j, t = t_j$$

Climatic hydrodynamic data for scenarios



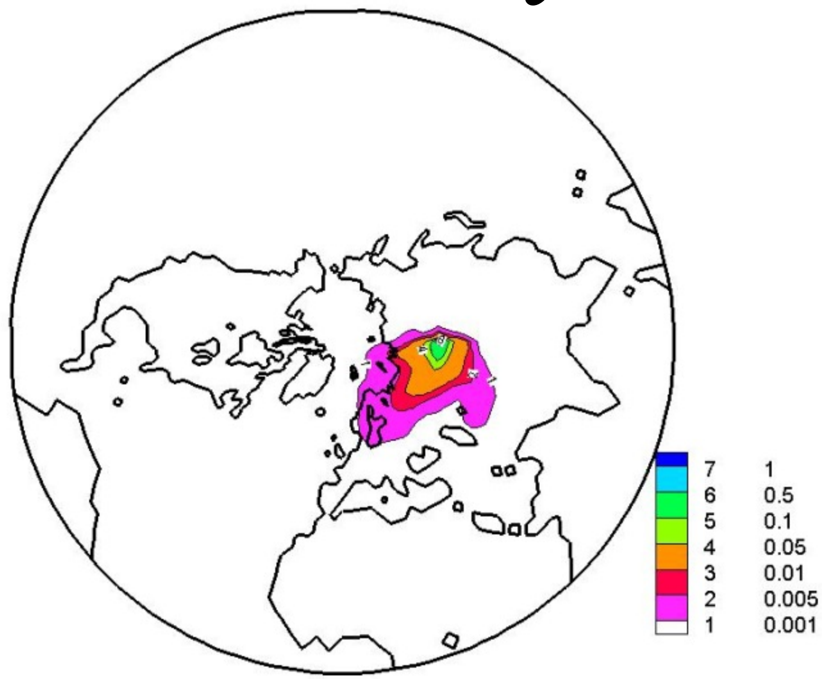
Fragment of the leading subspace for hgt500.

Inverse problem for risk/vulnerability assessment



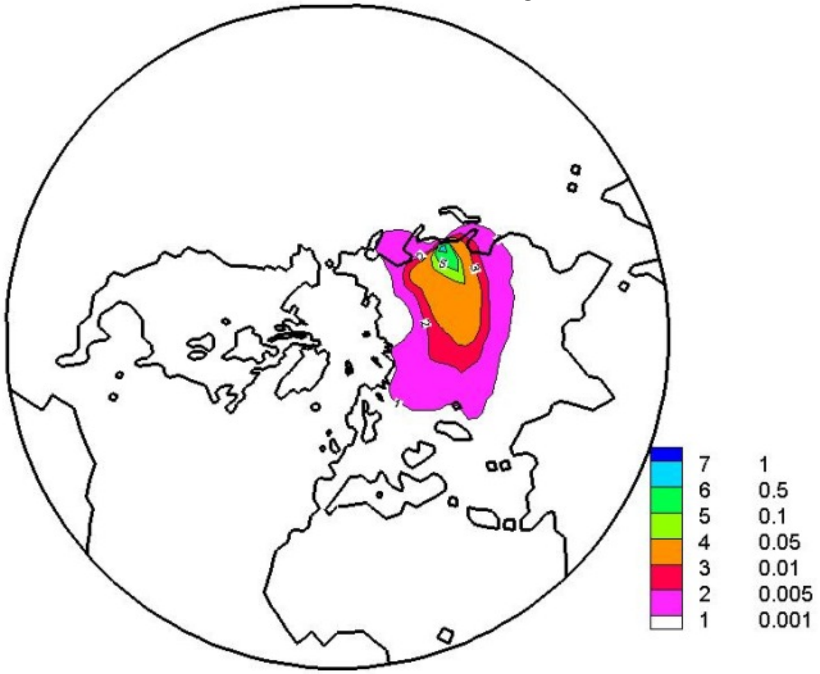
Prognostic assessments of climatically stipulated risks/vulnerability areas for receptor region - Ekaterinburg for October.

Inverse problem for risk/vulnerability assessment



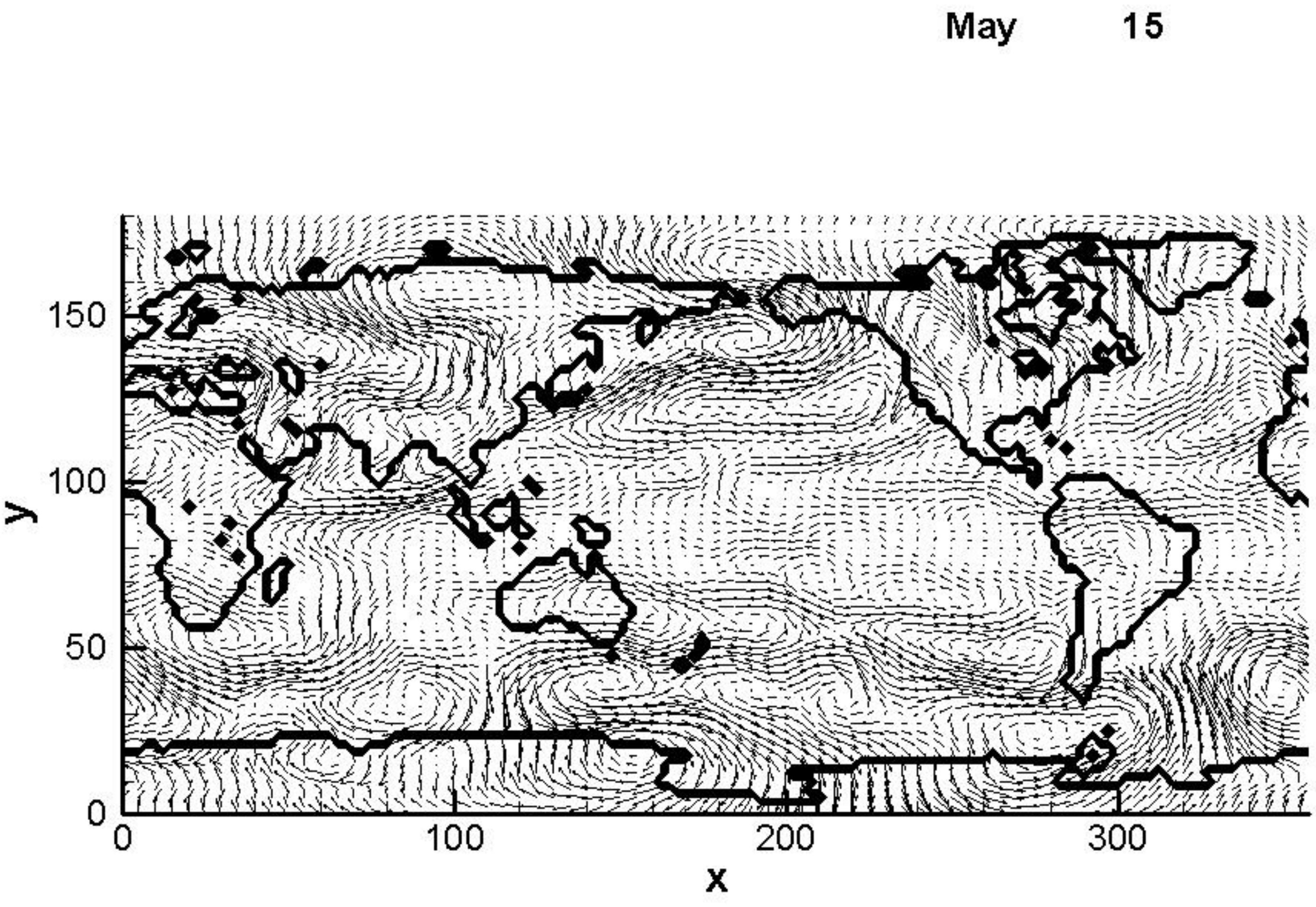
Prognostic assessments of climatically stipulated risks/vulnerability areas for Krasnoyarsk as receptor region for October.

Inverse problem for risk/vulnerability assessment

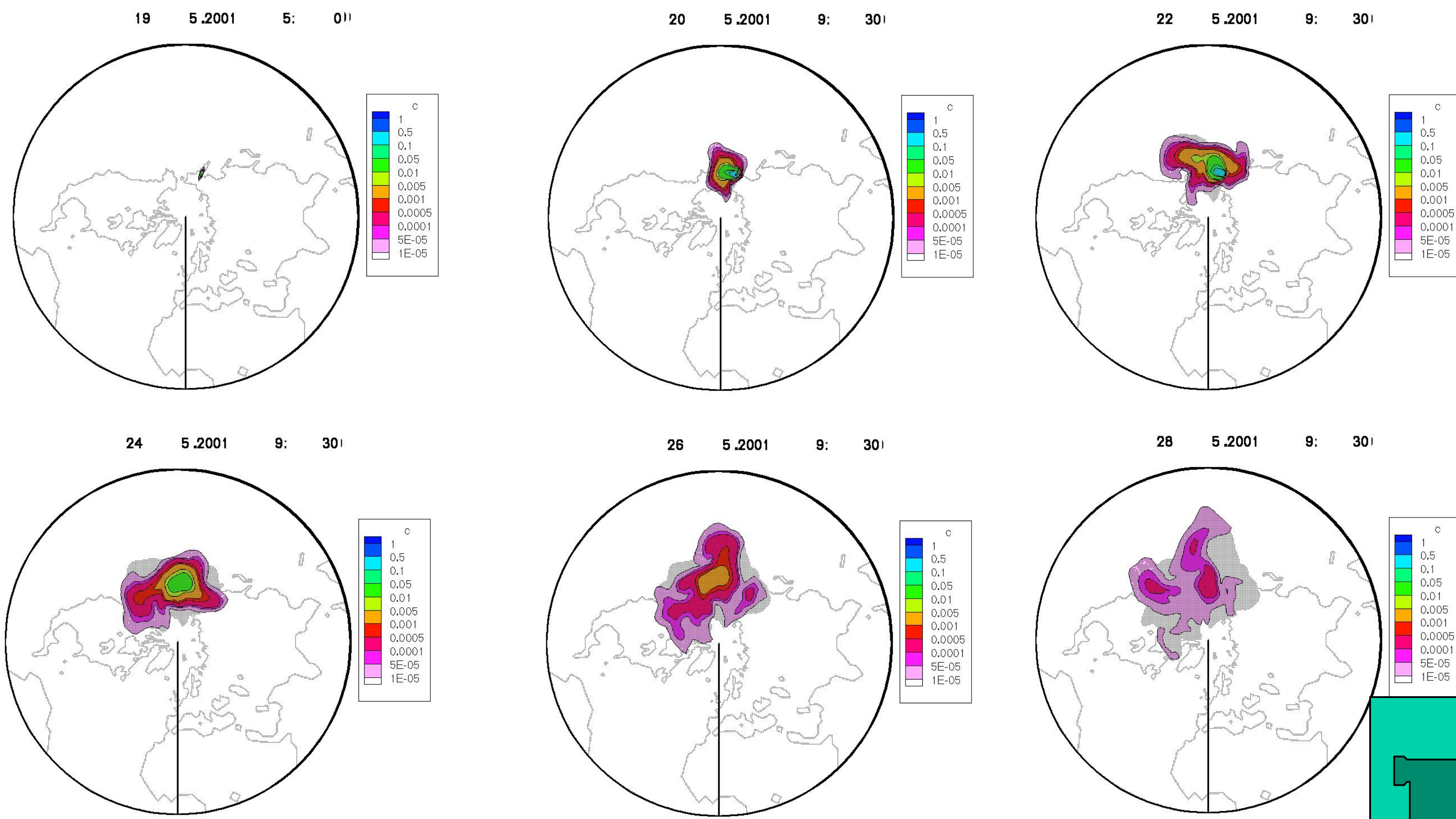


Prognostic assessments of climatically
stipulated risks/vulnerability areas
for Vladivostok as receptor region for October.

Climatic hydrodynamic data for scenarios

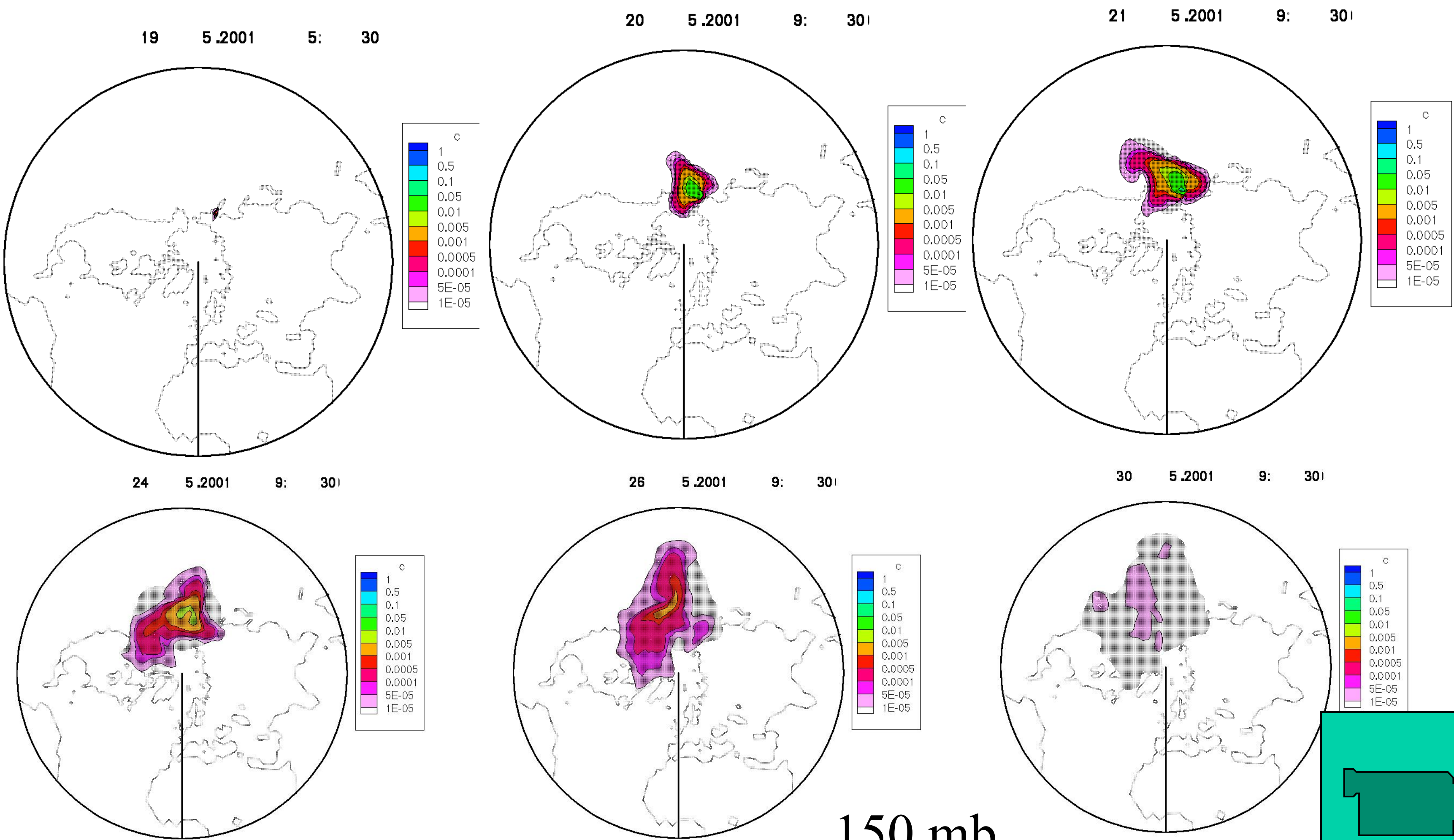


Volcano Schiveluch (Kamchatka, Russia) eruption 19-21.05.2001. Forward problem. Surface layer aerosol concentrations (<2 mkm)



animation

Volcano Schiveluch eruption
19-21.05.2001



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Conclusion

- Methodology and algorithms for studies of direct and feed-back relations are developed

- Methodology is intended for applications to environmental forecasting and design of nature-protection strategies

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Thank you for your time!